9. Arithmetic Progressions

Exercise 9.1

1. Question

Write the first five terms of each of the following sequences whose nth terms are :

(i) $a_n = 3n + 2$

(ii) $a_n = \frac{n-2}{3}$

(iii) $a_n = 3^n$

(iv) $a_n = \frac{3n-2}{5}$

(v) $a_n = (-1)^n . 2^n$

(vi) $a_n = \frac{n(n-2)}{2}$

(vii) $a_n = n^2 - n + 1$

(viii) $a_n = 2n^2 - 3n + 1$

(ix) $a_n = \frac{2n-3}{6}$

Answer

(i) $a_n = 3n + 2$

Put n = 1

A_1 = 3(1) + 2 = 3+2=5

Put n = 2

A_2 = 3(2) + 2=8

Put n = 3

A_3 = 3(3) + 2 = 9+2=11

Put n = 4

A_4 = 3(4) +2 = 12+2=14

Put n =5

$A_5 = 3(5) + 2 = 15+2=17$

(ii) $a_n = \frac{n-2}{3}$

Put n = 1

$A_1=\frac{1-2}{3} = \frac{-1}{3}$

Put n = 2

$A_2= \frac{2-2}{3} = 0$

Put n = 3

$A_3= \frac{3-2}{3} = \frac{1}{3}$

Put n = 4

$A_4= \frac{4-2}{3} = \frac{2}{3}$

Put n = 5

$A_5= \frac{5-2}{3} = \frac{3}{3} = 1$

(iii) $a_n = 3^n$

Put n = 1

A_1= 31= 3

Put n = 2

A_2= 32= 9

Put n = 3

A_3= 33= 27

Put n = 4

A_4= 34= 81

Put n = 5

A_5= 35= 243

(iv) $a_n = \frac{3n-2}{5}$

Put n = 1

$A_1= \frac{3(1)-2}{5} = \frac{3-2}{5} = \frac{1}{5}$

Put n = 2

$A_2 = \frac{3(2)-2}{5} = \frac{6-2}{5} = \frac{4}{5}$

Put n = 3

$A_3 = \frac{3(3)-2}{5} = \frac{9-2}{5} = \frac{7}{5}$

Put n = 4

$A_4 = \frac{3(4)-2}{5} = \frac{12-2}{5} = \frac{10}{5} = 2$

Put n = 5

$A_5 = \frac{3(5)-2}{5} = \frac{15-2}{5} = \frac{13}{5}$

(v) $a_n = (-1)^n.2^n$

Put n = 1

A_1= (-1)1.21 = (-1).2 = -2

Put n = 2

A_2= (-1)2.22 = (1).4 = 4

Put n = 3

A_3= (-1)3.23 = (-1).8 = -8

Put n = 4

A_4= (-1)4.24 = (1).16 = 16

Put n = 5

A_5= (-1)5.25 = (-1).32 = -32

(vi) $a_n = \frac{n(n-2)}{2}$

Put n = 1

$A_1 = \frac{1(1-2)}{2} = \frac{-1}{2}$

Put n = 2

$A_2 = \frac{2(2-2)}{2} = 0$

Put n = 3

$A_3 = \frac{3(3-2)}{2} = \frac{3}{2}$

Put n = 4

$A_4 = \frac{4(4-2)}{2} = \frac{8}{2} = 4$

Put n = 5

$A_5 = \frac{5(5-2)}{2} = \frac{15}{2}$

(vii) $a_n = n^2 - n + 1$

Put n = 1

$A_1 = (1)^2 - 1+1 = 1$

Put n = 2

$A_2 = (2)^2 -2+1 = 3$

Put n = 3

$A_3 = (3)^2 -3+1 = 9-2 = 7$

Put n = 4

$A_4 = (4)^2 -4+1 = 16-3 = 13$

Put n = 5

$A_5 = (5)^2 - 5+1 = 25-4 = 21$

(viii) $a_n = 2n^2 - 3n + 1$

Put n = 1

$A_1 = 2(1)^2 - 3(1) + 1 = 2-3+1 = 0$

Put n = 2

$A_2 = 2(2)^2 - 3(2) + 1 = 8-6+1 = 3$

Put n = 3

$A_3 = 2(3)^2 - 3(3) + 1 = 18-9+1 = 10$

Put n = 4

$A_4 = 2(4)^2 - 3(4) + 1 = 32-12+1 = 21$

Put n = 5

$A_5 = 2(5)^2 - 3(5) + 1 = 50-15+1 = 36$

(ix) $a_n = \frac{2n-3}{6}$

Put n = 1

$A_1 = \frac{2(1)-3}{6} = \frac{-1}{6}$

Put n = 2

$A_2 = \frac{2(2)-3}{6} = \frac{1}{6}$

Put n = 3

$A_3 = \frac{2(3)-3}{6} = \frac{3}{6} = \frac{1}{2}$

Put n = 4

$A_4 = \frac{2(4)-3}{6} = \frac{5}{6}$

Put n = 5

$A_5 = \frac{2(5)-3}{6} = \frac{7}{6}$

2. Question

Find the indicated terms in each of the following sequences whose nth terms are:

(i) $a_n = 5n - 4;\ a_{12}$ and a_{15}

(ii) $a_n = \frac{3n-2}{4n+5};\ a_7$ and a_8

(iii) $a_n = n(n-1)(n-2);\ a_5$ and a_8

(iv) $a_n = n(n-1)(2-n)(3+n);\ a_1, a_2, a_2$

(v) $a_n = (-1)^n n;\ a_3, a_5, a_8$

Answer

(i) $a_n = 5n - 4;\ a_{12}$ and a_{15}

Put n = 12

$A_{12} = 5(12) - 4$

= 60-4 = 56

Put n = 15

$A_{15} = 5(15) - 4$

= 75-4 = 71

(ii) $a_n = \frac{3n-2}{4n+5};\ a_7$ and a_8

Put n = 7

$A_7 = \frac{3(7)-2}{4(7)+5} = \frac{21-2}{28+5} = \frac{19}{33}$

Put n = 8

$A_8 = \frac{3(8)-2}{4(8)+5} = \frac{24-2}{32+5} = \frac{22}{37}$

(iii) $a_n = n(n-1)(n-2); a_5 \text{ and } a_8$

Put n = 5

$A_5 = 5(5-1)(5-2)$

$= 5(4)(3) = 60$

Put n = 8

$A_8 = 8(8-1)(8-2)$

$= 8(7)(6) = 336$

(iv) $a_n = n(n-1)(2-n)(3+n); a_1, a_2, a_2$

Put n = 1

$A_1 = 1(1-1)(2-1)(2-2)(3+2)$

$= 0$

Put n = 2

$A_2 = 2(2-1)(2-2)(3+2)$

$= 0$

Put n = 3

$A_3 = 3(3-1)(2-3)(3+3)$

$= 3(2)(-1)(6)$

$= 36$

(v) $a_n = (-1)^n n; a_3, a_5, a_8$

Put n = 3

$A_3 = (-1)^3 (3) = -3$

Put n = 5

$A_5 = (-1)^5 (5) = -5$

Put n = 8

$A_8 = (-1)^8 (8) = 8$

3. Question

Find the next five terms of each of the following sequences given by:

(i) $a_1 = 1, a_n = a_{n-1} + 2, n \geq 2$

(ii) $a_1 = a_2 = 2, a_n = a_{n-1} - 3, n > 2$

(iii) $a_1 = -1, a_n = \frac{a_{n-1}}{n}, n \geq 2$

(iv) $a_1 = 4, a_n = 4a_{n-1} + 3, n > 1$

Answer

(i) $a_1 = 1, a_n = a_{n-1} + 2, n \geq 2$

Put n = 2

$A_2 = a_{2-1} + 2$

$A_2 = a_1 + 2$

= 1+2 = 3

Put n = 3

$A_3 = a_{3-1} + 2$

$= a_2 + 2$

= 3+2 = 5

Put n = 4

$A_4 = a_{4-1} + 2$

= a3 + 2

= 5+2 = 7

Put n = 5

$A_5 = a_{5-1} + 2$

$= a_4 + 2$

= 7+2 = 9

Put n = 6

$A_6 = a_{6-1} + 2$

$= a_5 + 2$

= 9+2 = 11

(ii) $a_1 = a_2 = 2, a_n = a_{n-1} - 3, n > 2$

Put n = 3

$A_3 = a_{3-1} - 3$

$= a_2 - 3$

$= 2-3 = -1$

Put n = 4

$A_4 = a_{4-1} - 3$

$= a_3 - 3$

$= -1 - 3 = -4$

Put n = 5

$A_5 = a_{5-1} - 3$

$= a_4 - 3$

$= -4 - 3 = -7$

Put n = 6

$A_6 = a_{6-1} - 3$

$= a_5 - 3$

$= -7 - 3 = -10$

Put n = 7

$A_7 = a_{7-1} - 3$

$= a_6 - 3$

$= -10 - 3 = -13$

(iii) $a_1 = -1, a_n = \frac{a_{n-1}}{n}, n \geq 2$

Put n = 2

$A2 = a_{2-1} / 2$

$= -1/2$

Put n = 3

$A_3 = a_{3-1} / 3$

$= -1/6$

Put n = 4

$A_4 = a_{4-1} / 3$

$= -1/24$

Put n = 5

$A_5 = a_{5-1} / 3$

= -1/120

Put n = 6

$A_6 = a_{6-1} / 3$

= -1/720

(iv) $a_1 = 4, a_n = 4a_{n-1} + 3, n > 1$

Put n = 2

$A_2 = 4a_{2-1} + 3$

$= 4a_1 + 3 = 19$

Put n = 3

$A_3 = 4a_{3-1} + 3$

$= 4a_2 + 3 = 79$

Put n = 4

$A_4 = 4a_{4-1} + 3$

$= 4a_3 + 3$

= 316 + 3

= 319

Put n = 5

$A_5 = 4a_{5-1} + 3$

$= 4a_4 + 3$

= 1276 + 3

= 1279

Put n = 6

$A_6 = 4a_{6-1} + 3$

$= 4a_5 + 3$

= 5116 + 3

= 5119

Exercise 9.2

1. Question

For the following arithmetic progressions write the first term a and the common difference d :

(i) $-5, -1, 3, 7, \ldots$

(ii) $\frac{1}{5}, \frac{3}{5}, \frac{5}{5}, \frac{7}{5}, \ldots$

(iii) $0.3, 0.55, 0.80, 1.05, \ldots$

(iv) $-1.1, -3.1, -5.1, -7.1, \ldots$

Answer

(i) $-5, -1, 3, 7, \ldots$

First term, a = -5

Common difference, d = -1 – (-5)

= -1 + 5 = 4

(ii) $\frac{1}{5}, \frac{3}{5}, \frac{5}{5}, \frac{7}{5}, \ldots$

First term, a = $\frac{1}{5}$

Common difference, d = $\frac{3}{5} - \frac{1}{5} = \frac{2}{5}$

(iii) $0.3, 0.55, 0.80, 1.05, \ldots$

First term, a = 0.3

Common difference, d = 0.55 – 0.30 = 0.25

(iv) $-1.1, -3.1, -5.1, -7.1, \ldots$

First term, a = -1.1

Common difference, d = -3.1 – (-1.1)

= -2

2. Question

Write the arithmetic progression when first term a and common difference of d are as follows:

(i) $a = 4, \quad d = -3$

(ii) $a = -1, \quad d = \frac{1}{2}$

(iii) $a = -1.5, d = -0.5$

Answer

(i) $a = 4, \quad d = -3$

$a_1 = 4$, d= -3

$A_2 = a_1 + d = 4 - 3 = 1$

$A_3 = a_1 + 2d = 4 + 2(-3) = 4 - 6 = -2$

$A_4 = a_1 = 3d = 4 + 3(-3) = 4 - 9 = -5$

Series = 4, 1, -2, -5,...

(ii) $a = -1, \quad d = \frac{1}{2}$

$a_1 = -1$, $d = \frac{1}{2}$

$A_2 = a_1 + d = -1 + \frac{1}{2} = \frac{-1}{2}$

$A_3 = a_1 + 2d = -1 + 1 = 0$

$A_4 = a_1 + 3d = -1 + \frac{3}{2} = \frac{1}{2}$

Series = $-1, \frac{-1}{2}, 0, \frac{1}{2}, ...$

(iii) $a = -1.5, d = -0.5$

$a_1 = -1.5$, $d = -0.5$

$A_2 = a_1 + d = -1.5 - 0.5 = -2$

$A_3 = a_1 + 2d = -1.5 + 2(-0.5) = -1.5 - 1 = -2.5$

$A_4 = a_1 + 3d = -1.5 + 3(-0.5) = -1.5 - 1.5 = -3$

Series = $-1.5, -2, -2.5, -3, ...$

3. Question

In which of the following situation, the sequence of numbers formed will form an A.P.?

(i) The cost of digging a well for the first metre is Rs. 150 and rises by Rs.20 for each succeeding metre.

(ii)The mount of air present in the cylinder when a vacuum pump removes each time $\frac{1}{4}$ of their remaining in the cylinder.

Answer

(i) The cost of digging a well for the first metre = Rs. 150

The cost of digging a well for the second metre = 150 + 20 = Rs. 170

The cost of digging a well for the third metre = 170 + 20 = Rs. 190

The cost of digging a well for the fourth metre = 190 + 20 = Rs. 210

Difference between first and second metre = 170 – 150 = 20

Difference between second and third metre = 190 − 170 = 20

Difference between third and fourth metre = 210 − 190 = 20

Since, all the differences are equal. Therefore, it's an A.P.

(ii) Let the amount of air present in the cylinder first time = x

Amount of air present in the cylinder second time = $x - \frac{x}{4} = \frac{3x}{4}$

Amount of air present in the cylinder third time = $\frac{3x}{4} - \frac{3x}{16} = \frac{9x}{16}$

Difference in the amount of air present in the cylinder between first and the second time = $\frac{3x}{4} - x = \frac{-x}{4}$

Difference in the amount of air present in the cylinder between second and the third time = $\frac{9x}{16} - \frac{3x}{4}$

$= \frac{-3x}{16}$

Since, the differences are unequal. Therefore, it's not an A.P.

4. Question

Show that the sequence defined by $a_n = 5n - 7$ is an A.P., find its common difference.

Answer

Put n = 1

$A_1 = 5(1) - 7$

= -2

Put n = 2

$A_2 = 5(2) - 7$

= 10 − 7 = 3

Put n = 3

$A_3 = 5(3) - 7$

= 15 − 7 = 8

Common difference, $d_1 = a_2 - a_1 = 3 - (-2) = 5$

Common difference, $d_2 = a_3 - a_2 = 8 - 3 = 5$

Since, $d_1 = d_2$ and Common difference = 5

Therefore, it's an A.P.

5. Question

Show that the sequence defined by $a_n = 3n^2 - 5$ is not A.P.

Answer

Given: The sequence $a_n = 3n^2 - 5$.

To prove: the sequence defined by $a_n = 3n^2 - 5$ is not A.P.

Proof: Consider the sequence $a_n = 3n^2 - 5$,Put n = 1

$A_1 = 3(1)^2 - 5 = 3 - 5 = -2$

Put n = 2

$A_2 = 3(2)^2 - 5 = 12 - 5 = 7$

Put n = 3

$A_3 = 3(3)^2 - 5 = 27 - 5 = 22$In an A.P the difference of consecutive terms should be same.

So,

Common difference, $d_1 = a_2 - a_1 = 7 - (-2) = 9$

Common difference, $d_2 = a_3 - a_2 = 22 - 9 = 13$

Since, $d_1 \neq d_2$

Therefore, it's not an A.P.

6. Question

The general term of a sequence is given by $a_n = -4n^2 + 15$.Is the sequence an A.P..? If so, find its 15th term and the common difference.

Answer

Put n = 1

$A_1 = -4(1)^2 + 15 = 11$

Put n = 2

$A_2 = -4(2)^2 + 15 = -1$

Put n = 3

$A_3 = -4(3)^2 + 15 = -21$

Common difference, $d_1 = a_2 - a_1 = -1 - 11 = -12$

Common difference, $d_2 = a_3 - a_2 = -21 + 1 = 20$

Since, $d_1 \neq d_2$

Therefore, it's not an A.P.

7. Question

Find the common difference and write the next four terms of each of the following arithmetic progressions:

(i) $1, -2, -5, -8, \ldots$

(ii) $0, -3, -6, -9, \ldots$

(iii) $-1, \frac{1}{4}, \frac{3}{2}, \ldots$

(iv) $-1, -\frac{5}{6}, -\frac{2}{3}, \ldots$

Answer

(i) $1, -2, -5, -8, \ldots$

Common difference, d = -2 – 1 = -3

$A_5 = -8 - 3 = -11$

$A_6 = -11 - 3 = -14$

$A_7 = -14 - 3 = -17$

$A_8 = -17 - 3 = -20$

(ii) $0, -3, -6, -9, \ldots$

Common difference, d = -3 – 0 = -3

$A_5 = -9 - 3 = -12$

$A_6 = -12 - 3 = -15$

$A_7 = -15 - 3 = -18$

$A_8 = -18 - 3 = -21$

(iii) $-1, \frac{1}{4}, \frac{3}{2}, \ldots$

Common difference, d = $\frac{1}{4} + 1 = \frac{5}{4}$

$A_4 = \frac{3}{2} + \frac{5}{4} = \frac{11}{4}$

$A_5 = \frac{11}{4} + \frac{5}{4} = \frac{16}{4} = 4$

$A_6 = 4 + \frac{5}{4} = \frac{21}{4}$

$A_7 = \frac{21}{4} + \frac{5}{4} = \frac{26}{4} = \frac{13}{2}$

(iv) $-1, -\frac{5}{6}, -\frac{2}{3}, \ldots$

Common difference, d = $\frac{-5}{6} + 1 = \frac{1}{6}$

$A_4 = \frac{-2}{3} + \frac{1}{6} = \frac{-1}{2}$

$A_5 = \frac{-1}{2} + \frac{1}{6} = \frac{-1}{3}$

$A_6 = \frac{-1}{3} + \frac{1}{6} = \frac{-1}{6}$

$A_7 = \frac{-1}{6} + \frac{1}{6} = 0$

8. Question

Prove that no matter what is the real number a and b are, the sequence with nth term a+nb is always an A.P. What is the common difference?

Answer

Put n = 1

$A_1 = a + b$

Put n = 2

$A_2 = a + 2b$

Put n = 3

$A_3 = a + 3b$

Put n = 4

$A_4 = a + 4b$

Common difference, $d_1 = a_2 - a_1 = a + 2b - a - b = b$

Common difference, $d_2 = a_3 - a_2 = a + 3b - a - 2b = b$

Common difference, $d_3 = a_4 - a_3 = a + 4b - a - 3b = b$

Since, $d_1 = d_2 = d_3$

Therefore, it's an A.P. with common difference 'b'

9. Question

Write the sequence with nth term:

(i) $a_n = 3 + 4n$

(ii) $a_n = 5 + 2n$

(iii) $a_n = 6 - n$

(iv) $a_n = 9 - 5n$

Show that all of the above sequences form A.P.

Answer

(i) Put n = 1

$A_1 = 3 + 4(1) = 7$

Put n = 2

$A_2 = 3 + 4(2) = 11$

Put n = 3

$A_3 = 3 + 4(3) = 15$

Common difference, $d_1 = a_2 - a_1 = 11 - 7 = 4$

Common difference, $d_2 = a_3 - a_2 = 15 - 11 = 4$

Since, $d_1 = d_2$

Therefore, it's an A.P. with sequence 7, 11, 15,...

(ii) Put n = 1

$A_1 = 5 + 2(1) = 7$

Put n = 2

$A_2 = 5 + 2(2) = 9$

Put n = 3

$A_3 = 5 + 2(3) = 11$

Common difference, $d_1 = a_2 - a_1 = 9 - 7 = 2$

Common difference, $d_2 = a_3 - a_2 = 11 - 9 = 2$

Since, $d_1 = d_2$

Therefore, it's an A.P. with sequence 7, 9, 11,...

(iii) Put n = 1

$A_1 = 6 - 1 = 5$

Put n = 2

$A_2 = 6 - 2 = 4$

Put n = 3

$A_3 = 6 - 3 = 3$

Common difference, $d_1 = a_2 - a_1 = 4 - 5 = -1$

Common difference, $d_2 = a_3 - a_2 = 3 - 4 = -1$

Since, $d_1 = d_2$

Therefore, it's an A.P. with sequence 5, 4 , 3,...

(iv) Put n = 1

$A_1 = 9 - 5(1) = 4$

Put n = 2

$A_2 = 9 - 5(2) = -1$

Put n = 3

$A_3 = 9 - 5(3) = -6$

Common difference, $d_1 = a_2 - a_1 = -1 - 4 = -5$

Common difference, $d_2 = a_3 - a_2 = -6 - (-1) = -5$

Since, $d_1 = d_2$

Therefore, it's an A.P. with sequence 4, -1, -6,...

10. Question

Find out which of the following sequences are arithmetic progressions. For those which are arithmetic progressions, find out the common difference.

(i) $3, 6, 12, 24, \ldots$

(ii) $0, -4, -8, -12, \ldots$

(iii) $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \ldots$

(iv) $12, 2, -8, -18, \ldots$

(v) $3, 3, 3, 3, \ldots$

(vi) $p, p+90, p+180, p+270 \ldots where\ p = (999)^{999}$

(vii) $1.0, 1.7, 2.4, 3.1, \ldots$

(viii) $-225, -425, -625, -825, \ldots$

(ix) $10, 10+2^5, 10+2^7$

(x) $a+b, (a+1)+b, (a+1)+(b+1), (a+2)+(b+1), (a+2)+(b+2), \ldots$

(xi) $1^2, 3^2, 5^2, 7^2, \ldots$

(xii) $1^2, 5^2, 7^2, 73, \ldots$

Answer

(i) $3, 6, 12, 24, \ldots$

Common difference, $d_1 = 6 - 3 = 3$

Common difference, $d_2 = 12 - 6 = 6$

Since, $d_1 \neq d_2$

Therefore, it's not an A.P.

(ii) $0, -4, -8, -12, \ldots$

Common difference, $d_1 = -4 - 0 = -4$

Common difference, $d_2 = -8 - (-4) = -4$

Since, $d_1 = d_2$

Therefore, it's an A.P. with common difference, $d = -4$

(iii) $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \ldots$

Common difference, $d_1 = \frac{1}{4} - \frac{1}{2} = \frac{-1}{2}$

Common difference, $d_2 = \frac{1}{6} - \frac{1}{4} = \frac{-1}{12}$

Since, $d_1 \neq d_2$

Therefore, it's not an A.P.

(iv) $12, 2, -8, -18, \ldots$

Common difference, $d_1 = 2 - 12 = -10$

Common difference, $d_2 = -8 - 2 = -10$

Since, $d_1 = d_2$

Therefore, it's an A.P. with common difference, $d = -10$

(v) $3, 3, 3, 3, \ldots$

Common difference, $d_1 = 3 - 3 = 0$

Common difference, $d_2 = 3 - 3 = 0$

Since, $d_1 = d_2$

Therefore, it's an A.P. with common difference, $d = 0$

(vi) $p, p + 90, p + 180, p + 270 \ldots \text{where } p = (999)^{999}$

Common difference, $d_1 = p + 90 - p = 90$

Common difference, d_2= p + 180 − p − 90 = 90

Since, d_1=d_2

Therefore, it's an A.P. with common difference, d = 90

(vii) $1.0, 1.7, 2.4, 3.1, \ldots$

Common difference, d_1= 1.7 − 1.0 = 0.7

Common difference, d_2= 2.4 − 1.7 = 0.7

Since, d_1=d_2

Therefore, it's an A.P. with common difference, d = 0.7

(viii) $-225, -425, -625, -825, \ldots$

Common difference, d_1= -425 + 225 = -200

Common difference, d_2= -625 + 425 = -200

Since, d_1=d_2

Therefore, it's an A.P. with common difference, d = -200

(ix) $10, 10+2^6, 10+2^7$

Common difference, d_1= 10 + 2^6 − 10 = 2^6 = 64

Common difference, d_2 = 10 + 2^7 − 10 − 2^6 = 2^6 (2 − 1) = 64

Since, d_1=d_2

Therefore, it's an A.P. with common difference, d = 64

(x) $a+b, (a+1)+b, (a+1)+(b+1), (a+2)+(b+1), (a+2)+(b+2), \ldots$

Common difference, d_1 = (a + 1) + b − a − b = 1

Common difference, d_2 = (a + 1) + (b + 1) − (a + 1) − b = 1

Since, d_1 = d_2

Therefore, it's an A.P. with common difference, d = 1

(xi) $1^2, 3^2, 5^2, 7^2, \ldots$

Common difference, d_1= 3^2 − 1^2 = 8

Common difference, d_2 = 5^2 − 3^2 = 25 − 9 = 16

Since, $d_1 \neq d_2$

Therefore, it's not an A.P.

(xii) $1^2, 5^2, 7^2, 73, \ldots$

Common difference, $d_1 = 5^2 - 1^2 = 24$

Common difference, $d_2 = 7^2 - 5^2 = 24$

Since, $d_1 = d_2$

Therefore, it's an A.P. with common difference, d = 24

11. Question

Find the common difference of the A.P. and write the next two terms:

(i) $51, 59, 67, 75, \ldots$

(ii) $75, 67, 59, 51, \ldots$

(iii) $1.8, 2.0, 2.2, 2.4, \ldots$

(iv) $0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \ldots$

(v) $119, 136, 153, 170, \ldots$

Answer

(i) $51, 59, 67, 75, \ldots$

Common difference, d = 59 – 51 = 8

$A_5 = 75 + 8 = 83$

$A_6 = 83 + 8 = 91$

(ii) $75, 67, 59, 51, \ldots$

Common difference, d = 67 – 75 = -8

$A_5 = 51 - 8 = 43$

$A_6 = 43 - 8 = 35$

(iii) $1.8, 2.0, 2.2, 2.4, \ldots$

Common difference, d = 2.0 – 1.8 = 0.2

$A_5 = 2.4 + 0.2 = 2.6$

$A_6 = 2.6 + 0.2 = 2.8$

(iv) $0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \ldots$

Common difference, $d = \frac{1}{4} - 0 = \frac{1}{4}$

$A_5 = \frac{3}{4} + \frac{1}{4} = 4$

$A_6 = 4 + \frac{1}{4} = \frac{5}{4}$

(v) 119, 136, 153, 170, ...

Common difference, d = 136 − 119 = 17

$A_5 = 170 + 17 = 187$

$A_6 = 187 + 17 = 204$

12. Question

The nth term of an A.P. is 6n+2. Find the common difference.

Answer

$a_n = 6n + 2$

Put n = 1

$A_1 = 6(1) + 2 = 8$

Put n = 2

$A_2 = 6(2) + 2 = 14$

Common difference, $d = a_2 - a_1 = 14 - 8 = 6$

Exercise 9.3

1. Question

Find:

(i) 10^{th} term of the A.P. 1, 4, 7, 10, ...

(ii) 18^{th} term of the A.P. $\sqrt{2}, 3\sqrt{2}, 5\sqrt{2}, \ldots$

(iii) n^{th} term of the A.P. 13, 8, 3, −2, ...

(iv) 10^{th} term of the A.P. −40, −15, 10, 35, ...

(v) 8^{th} term of the A.P. 117, 104, 91, 78, ...

(vi) 11^{th} term of the A.P. 10.0, 10.5, 11.0, 11.5, ...

(vii) 9^{th} term of the A.P. $\frac{3}{4}, \frac{5}{4}, \frac{7}{4}, \frac{9}{4}, \ldots$

Answer

(i) 10^{th} term of the A.P. 1, 4, 7, 10, ...

a = 1, d = 4 − 1 = 3

$A_{10} = a + (10 - 1)\,d$

$= 1 + (9)3 = 28$

(ii) 18^{th} term of the A.P. $\sqrt{2}, 3\sqrt{2}, 5\sqrt{2}, \ldots$

$a = \sqrt{2}$, $d = 3\sqrt{2} - \sqrt{2} = 2\sqrt{2}$

$A_{18} = a + (18 - 1)\,d$

$= \sqrt{2} + (17)\, 2\sqrt{2} = 35\sqrt{2}$

(iii) n^{th} term of the A.P. $13, 8, 3, -2, \ldots$

$a = 13$, $d = -5$

$A_n = a + (n-1)d = 13 + (n-1)(-5)$

$= 13 - 5n + 5 = 18 - 5n$

(iv) 10^{th} term of the A.P. $-40, -15, 10, 35, \ldots$

$a = -40$, $d = 25$

$A_{10} = a + (10-1)\,d = -40 + 9\,(25)$

$= -40 + 225 = 185$

(v) 8^{th} term of the A.P. $117, 104, 91, 78, \ldots$

$a = 117$, $d = -13$

$A_8 = a + (8 - 1)\,d = 117 + 7\,(-13)$

$= 117 - 91 = 26$

(vi) 11^{th} term of the A.P. $10.0, 10.5, 11.0, 11.5, \ldots$

$a = 10.0$, $d = 10.5 - 10.0 = 0.5$

$A_{11} = a + (11 - 1)\,d = 10.0 + 10\,(0.5)$

$= 10.0 + 5 = 15.0$

(vii) 9^{th} term of the A.P. $\frac{3}{4}, \frac{5}{4}, \frac{7}{4}, \frac{9}{4}, \ldots$

$a = \frac{3}{4}$, $d = \frac{5}{4} - \frac{3}{4} = \frac{1}{2}$

$A_9 = a + (9 - 1)\,d = \frac{3}{4} + 8\,(\frac{1}{2}) = \frac{19}{4}$

2 A. Question

(i) Which term of the A.P. $3, 8, 13, \ldots$ is 248?

Answer

$a = 3$, $d = 5$

$A_n = a + (n - 1) d$

$248 = 3 + (n - 1) 5 = 3 + 5n - 5$

$248 = -2 + 5n$

$250 = 5n$

$n = 50$

2 B. Question

Which term of the A.P. $84, 80, 76 \ldots$ is 0?

Answer

$a = 84$, $d = -4$, $A_n = 0$

We know, $A_n = a + (n - 1) d$

$0 = 84 + (n - 1) -4$

$0 = 84 - 4n + 4$

$0 = 88 - 4n$

$n = 22$

2 C. Question

Which term of the A.P. $4, 9, 14, \ldots$ is 254?

Answer

$a = 4$, $d = 5$

$A_n = a + (n - 1) d$

$254 = 4 + (n - 1) 5$

$254 = 4 + 5n - 5$

$254 = -1 + 5n$

$255 = 5n$

$n = 51$

2 D. Question

Which term of the A.P. $21, 42, 63, 84, \ldots$ is 420?

Answer

$a = 21$, $d = 21$

$A_n = a + (n - 1) d$

$420 = 21 + (n - 1) 21$

$420 = 21 + 21n - 21$

$420 = 21n$

$n = 20$

2 E. Question

Which term of the A.P. $121, 117, 113, \ldots$ is its first negative term?

Answer

$a = 121$, $d = -4$

$A_n = -ve$

$A_n = a + (n - 1) d$

$= 121 + (n - 1) -4$

$= 125 - 4n$

Since, we want –ve term

Therefore, 4n should be a number greater than 125

Hence, 128 is the number greater than 125 and divisible by 4

$4n = 128$

$n = 32$

Therefore, 32^{nd} term of the given A.P. is –ve

3 A. Question

Is 68 a term of the A.P. $7, 10, 13, \ldots$?

Answer

$a = 7$, $d = 3$

$A_n = a + (n - 1) d$

$68 = 7 + (n - 1) 3$

$68 = 4 + 3n$

$3n = 64$

Since, 64 is not divisible by 3

Therefore, 68 isn't the term of the given A.P.

3 B. Question

Is 302 a term of the A.P. $3, 8, 13, \ldots$?

Answer

a = 3, d = 5

A_n= a + (n − 1) d

302 = 3 + (n − 1) 5

302 = -2 + 5n

304 = 5n

Since, 304 is not divisible by 5

Therefore, 302 isn't the term of the given A.P.

3. Question

Is -150 a term of the A.P. $11, 8, 5, 2, \ldots$?

Answer

a = 11, d = -3

A_n= a + (n − 1) d

-150 = 11 + (n − 1) (-3)

-150 = 11 − 3n + 3

-150 = 14 − 3n

-164 = 3n

Since, -164 is not divisible by 3

Therefore, -150 isn't the term of the given A.P.

4. Question

How many terms are there in the A.P.?

(i) $7, 10, 13, \ldots 43.$

(ii) $-1, -\frac{5}{6}, -\frac{2}{3}, -\frac{1}{2}, \ldots, \frac{10}{3}.$

(iii) $7, 13, 19, \ldots, 205.$

(iv) $18, 15\frac{1}{2}, 13, \ldots, -47$

Answer

(i) $7, 10, 13, \ldots 43.$

a = 7, d = 3

A_n = 43

a + (n − 1) d = 43

$7 + (n - 1)\ 3 = 43$

$3n = 39$

$n = 13$

Therefore, there are total 13 terms in the A.P.

(ii) $-1, -\frac{5}{6}, -\frac{2}{3}, -\frac{1}{2}, \ldots, \frac{10}{3}$.

$a = -1,\ d = \frac{1}{6}$

$A_n = \frac{10}{3}$

$a + (n - 1)\ d = \frac{10}{3}$

$-1 + (n - 1)\ \frac{1}{6} = \frac{10}{3}$

$n = 26$

(iii) $7, 13, 19, \ldots, 205$.

$a = 7,\ d = 6$

$A_n = 205$

$a + (n - 1)\ d = 205$

$7 + (n - 1)\ 6 = 205$

$6n = 204$

$n = 34$

(iv) $18, 15\frac{1}{2}, 13, \ldots, -47$

$a = 18,\ d = \frac{-5}{2}$

$A_n = -47$

$a + (n - 1)\ d = -47$

$18 + (n - 1)\ \frac{-5}{2} = -47$

$18 - \frac{5n}{2} + \frac{5}{2} = -47$

$n = 27$

5. Question

The first term of an A.P. is 5, the common difference is 3 and the last term is 80; find the number of terms.

Answer

Given: The first term of an A.P. is 5, the common difference is 3 and the last term is 80**To find:** the number of terms.**Solution :**We have a = 5, d = 3

a_n = 80From the formula $a_n = a + (n-1)d$ Number of terms is : $n = \frac{a_n - a}{d} + 1$

$= \frac{(80-5)}{3} + 1$

$= \frac{75}{3} + 1$

= 25 + 1

= 26Hence the number of terms in a given A.P. is 26.

6. Question

The 6^{th} and 17^{th} terms of an A.P. are 19 and 41 respectively, find the 40^{th} term.

Answer

$a_6 = 19 = a + (n - 1) d$

=19 = a + 5d …(i)

$A_{17} = 41 = a + (n - 1) d$

= 41 = a + 16d …(ii)

Subtracting (i) from (ii), we get

22 = 11d

d = 2

Now substituting the value of d in (i): 19 = a + 10

= a = 9

$A_{40} = a + (40 - 1) 2$

= 9 + 78

= 87

7. Question

If 9^{th} term of an A.P. is zero, prove that its 29^{th} term is double the 19^{th} term.

Answer

$a_9 = 0$

a + (9 – 1)d =0

a + 8d = 0

$a = -8d$ …(i)

To Prove: $a_{29} = 2a_{19}$

Proof: LHS= $a_{29} = a + 28d = -8d + 28d = 20d$

RHS= $2a_{19} = 2[a + (18)d] = 2(-8d + 18d) = 2(10d) = 20d$

Since, LHS=RHS

Hence, proved

8. Question

If 10 times the 10^{th} term of an A.P. is equal to 15 times the 15^{th} term, show that 25^{th} term of the A.P. is zero.

Answer

$10a_{10} = 15a_{15}$

$10[a + (10 - 1)d] = 15[a + (15 - 1)d]$

$2a + 2(9)d = 3a + 3(14)d$

$-a = 42d - 18d$

$-a = 24d$

$a = -24d$

$a_{25} = a + 24d$

$= -24d + 24d$

$= 0$

9. Question

The 10^{th} and 18^{th} terms of an A.P. are 41 and 73 respectively. Find 26^{th} term.

Answer

$a_{10} = 41$

$a + (10 - 1)d = 41$

$a + 9d = 41$ …(i)

$a_{18} = 73$

$a + (18 - 1)d = 73$

Substituting the value of a from (i),

$41 - 9d + 17d = 73$

$8d = 32$

$d = 4$

$a = 41 - 9(4)$

$a = 41 - 36$

$a = 5$

$a_{26} = a + (26 - 1)d$

$= 5 + 25(4)$

$= 5 + 100$

$= 105$

10. Question

In a certain A.P. the 24^{th} term is twice the 10^{th} term. Prove that the 72^{nd} term is twice the 34^{th} term.

Answer

Given: $a_{24} = 2a_{10}$

To Prove: $a_{72} = 2a_{34}$

Proof: $a_{24} = 2a_{10}$

$a + (24 - 1)d = 2a + 2(10 - 1)d$

$23d - 18d = a$

$a = 5d$

LHS= $a_{72} = a + 71d = 5d + 71d = 76d$

RHS= $a_{34} = 2a + 2(33)d = 10d + 66d = 76d$

Since, LHS=RHS

Hence, proved

11. Question

If $(m+1)^{th}$ term of an A.P. is twice the $(n+1)^{th}$ term, prove that $(3m+1)^{th}$ term is twice $(m+n+1)^{th}$ term.

Answer

Given: $(m+1)^{th}$ term of an A.P. is twice the $(n+1)^{th}$ term$a_{(m+1)} = 2a_{(n+1)}$

To Prove:$(3m+1)^{th}$ term is twice $(m+n+1)^{th}$ term $a_{(3m+1)} = 2a_{(m+n+1)}$

Proof: $a_{(m+1)} = 2a_{(n+1)}$

$\Rightarrow a + (m + 1 - 1)d = 2a + 2(n + 1 - 1)d$

$\Rightarrow -a = 2nd - md$

⇒ a = md- 2nd (i)

LHS: a_{3m+1}= a + (3m +1 -1)d = md – 2nd + 3md = 2d(2m - n)

RHS: 2$a_{(m + n + 1)}$ = 2[a +(m +n +1 -1)d]= 2[md -2nd +md +nd] = 2d(2m - n)

LHS = RHS

Hence, proved

12. Question

If the n^{th} term of the A.P. 9, 7, 5,... is same as the n^{th} term of the A.P. 15, 12, 9,...find n.

Answer

A=9, D=7-9=-2

a=15, d=-3

A_n=a_n

A+(n -1)D=a +(n -1)d

9 – (n -1)2=15 – (n -1)3

(n -1)(3 -2)=6

n -1=6

n=7

13. Question

Find the 12^{th} term from the end of the following arithmetic progressions:

(i) 3, 5, 7, 9, ...201

(ii) 3, 8, 13, ..., 253

(iii) 1, 4, 7, 10, ..., 88

Answer

(i) 3, 5, 7, 9,...201

a=3, d=5 -3=2, a_n =201

a_n= a +(n -1)d

201 =3 +2n – 2

N =100

Now, we have to find 12^{th} term from the last that means,

100^{th} – 11=89^{th} term

Then,

$a_{89} = a + (89 - 1)d$

$= 3 + 88 \times 2$

$= 179$

Hence, the 12^{th} term from the end of the A.P. is 179.

(ii) $3, 8, 13, \ldots, 253$

$a = 3$, $d = 8 - 3 = 5$

$a_n = 253$

$a + (n - 1)d = 253$

$3 + (n - 1)5 = 253$

$n = 51$

Now, we have to find 12^{th} term from the last that means,

51th -11 = 40^{th} term

Then,

$a_{40} = a + (n - 1)d$

$= 3 + 39(5)$

$= 198$

Hence, the 12^{th} term from the end of the A.P. is 198

(iii) $1, 4, 7, 10, \ldots, 88$

$a = 1$, $d = 4 - 1 = 3$

$a_n = 88$

$a + (n - 1)d = 88$

$1 + (n - 1)3 = 88$

$n = 30$

Now, we have to find 12^{th} term from the last that means,

$30^{th} - 11 = 19^{th}$ term

$a_{19} = a + 18d$

$= 1 + 18(3)$

$= 55$

Hence, the 12^{th} term from the end of the A.P. is 198.

14. Question

The 4^{th} term of an A.P. is three times the first and the 7^{th} term exceeds twice the third term by 1. Find the first term and the common difference.

Answer

Given:

$a_4 = 3a_1$ ...(i)

$a_7 = 2a_3 + 1$ ...(ii)We know $a_n = a + (n-1)d$

So,

$a_3 = a + 2d$

$a_4 = a + 3d$

$a_7 = a + 6d$

Put all the values of a_1 and a_4 in (i),

$a + 3d = 3a$

$\Rightarrow 3d = 3a - a \Rightarrow 3d = 2a$

$\Rightarrow d = \frac{2a}{3}$

Put all the values of a_7 and a_3 in (ii),

$\Rightarrow a + 6d = 2(a + 2d) + 1 \Rightarrow a + 6d = 2a + 4d + 1$

$\Rightarrow 6d - 4d = 2a - a + 1$

$\Rightarrow 2d = a + 1$

Put $d = \frac{2a}{3}$,

$\Rightarrow 2(\frac{2a}{3}) = a + 1$

$\Rightarrow \frac{4a}{3} = a + 1$

$$\Rightarrow \frac{4a}{3} - a = 1$$

$$\Rightarrow \frac{4a - 3a}{3} = 1$$

$$\Rightarrow \frac{a}{3} = 1$$

$\Rightarrow a = 3$

Now, difference, d= $\frac{2a}{3}$

$$\Rightarrow d = \frac{2(3)}{3}$$

$$d = \frac{6}{3}$$

⇒ d= 2

Hence, first term is 3 and the common difference is 2.

15. Question

Find the second term and n^{th} term of an A.P. whose 6^{th} term is 12 and the 8^{th} term is 22.

Answer

a_6= a + 5d

12= a + 5d ...(i)

a_8= a + 7d

22= a + 7d ...(ii)

Subtracting (i) from (ii), we get,

10= 2d

d= 5

from (i)

12 = a + 5(5)

12= a + 25

a= -13

a_2= a +d

=-13 +5

=-8

a_n= a + (n-1)d

= -13 + (n-1)5

=-18 +5n

16. Question

How many numbers of two digit are divisible by 3?

Answer

Two digit numbers divisible by 3 are 12, 15, 18,... ,99

Hence, a= 12, ,d=3

a_n= a + (n -1)d

99= 12(n -1)3

99= 9 = 3n

n=30

Hence, the total numbers of two digit divisible by 3 are 30.

17. Question

An A.P. consists of 60 terms. If the first and the last terms be 7 and 125 respectively, find 32^{nd} term.

Answer

n =60, a=7, a_{60} =l =125

a_{60} = a + 59d

125 = 7 + 59d

d= $\frac{118}{59}$

d=2

a_{32}= a + 31d

= 7 + 31(2)

=69

Hence, 32th term is 69 in the given A.P.

18. Question

The sum of 4^{th} and 8^{th} terms of an A.P. is 24 and the sum of the 6^{th} and 10^{th} terms is 34. Find the first term and the common difference of the A.P.?

Answer

Given: $a_4 + a_8$ = 24 ...(i)

$a_6 + a_{10}$ = 34 ...(ii)

a_4 = a + 3d

a_8 =a + 7d

a_6 =a + 5d

a_{10}=a + 9d

Put the value of a_4 and a_8 in (i)

(a + 3d) + (a + 7d) = 24

a + 5d =12 ...(iii)

Put the value of a_6 and a_{10} in (ii)

(a +5d) + (a +9d) =34

a +7d =17 ...(iv)

Subtracting (iii) from (iv), we get

2d = 5

d = $\frac{5}{2}$

Putting the value of d in (iii), we get

a = 12 - $\frac{25}{2} = \frac{-1}{2}$

Hence, first term is $\frac{-1}{2}$ and common difference is $\frac{5}{2}$

19. Question

The first term of an A.P. is 5 and its 100th term is - 292. Find the 50th term of this A.P.

Answer

Given, a = 5

a_{100}= -292

5 − 99d = -292

d = $\frac{-297}{-99}$ = 3

Now, 50th term, a_{50} = a + (50 − 1) d

= 5 + (49) 3 = 5 + 147 = 152

Hence, 50th term of given A.P. is 152

20. Question

Find $a_{30} - a_{20}$ for the A.P.

(i) $-9, -14, -19, -24, \ldots$

(ii) $a, a+d, a+2d, a+3d, \ldots$

Answer

(i) $-9, -14, -19, -24, \ldots$

a = -9, d = -14 + 9 = -5

$a_{30} = a + 29d$

$= -9 + 29(-5) = -154$

$a_{20} = a + 19d$

$= -9 + 19(-5) = -104$

$a_{30} - a_{20} = -154 - (-104)$

$= -154 + 104$

$= -50$

(ii) $a, a+d, a+2d, a+3d, \ldots$

a = a, d = a + d – a = d

$a_{30} = a + 29d$

$a_{20} = a + 19d$

$a_{30} - a_{20} = a + 29d - a - 19d$

$= 10d$

21. Question

Write the expression $a_n - a_k$ for the A.P. $a, a+d, a+2d, \ldots$

Hence, find the common difference of the A.P. for which

(i) 11^{th} terms is 5 and 13^{th} term is 79.

(ii) $a_{10} - a_5 = 200$

(iii) 20^{th} term is 10 more than the 18^{th} term.

Answer

Let n^{th} term $a_n = a + (n - 1)d$

$= a + nd - d$

k^{th} term, $a_k = a + (k - 1)d$

$= a + kd - d$

Now,

$a_n - a_k = (a + nd - d) - (a + kd - d)$

$= nd - kd = d(n - k)$

(i) $a_{11} = 5$

$a + 10d = 5$ (i)

$a_{13} = 79$

a + 12d = 79 (ii)

By subtracting (i) from (ii), we get

2d = 74

d = 37

Hence, the common difference is 37

(ii)a_{10} = a + 9d (i)

a_5 = a + 4d (ii)

$a_{10} - a_5$ = a + 9d − a − 4d

200 = 5d

d = 40

Hence, common difference is 40

(iii) a_{20} = a + 19d (i)

a_{18} = a + 17d (ii)

Given that $a_{20} = a_{18} + 10$

a + 19d = a + 17d + 10

2d = 10

d = 5

Hence, common difference is 5

22. Question

Find n if the given value of x is the n^{th} term of the given A.P.

(i) $25, 50, 75, 100, \ldots; x = 1000$

(ii) $-1, -3, -5, -7, \ldots; x = -151$

(iii) $5\frac{1}{2}, 11, 16\frac{1}{2}, 22, \ldots; x = 550$

(iv) $1, \frac{21}{11}, \frac{31}{11}, \frac{41}{11}, \ldots, x = \frac{171}{11}$

Answer

(i) $25, 50, 75, 100, \ldots; x = 1000$

a = 25, d = 50 − 25 = 25

Last term, l = 1000

Number of terms, n = $\frac{l-a}{d}$ + 1

= $\frac{1000-25}{25}$ + 1 = 40

Hence, the value of n is 40

(ii) $-1, -3, -5, -7, \ldots; x = -151$

a = -1, d = -2

Last term, l = -151

Number of terms, n = $\frac{l-a}{d}$ + 1

= $\frac{-151-(-1)}{-2}$ + 1 = 76

Hence, the value of n is 76

(iii) $5\frac{1}{2}, 11, 16\frac{1}{2}, 22, \ldots; x = 550$

a = $\frac{11}{2}$, d = $\frac{11}{2}$

Last term, l = 550

l = a + (n − 1) d

550 = $\frac{11}{2}$ + (n − 1) $\frac{11}{2}$

550 = $\frac{11n}{2}$

1100 = 11n

n = 100

Hence, number of terms, n = 100

(iv) $1, \frac{21}{11}, \frac{31}{11}, \frac{41}{11}, \ldots, x = \frac{171}{11}$

a = 1, d = $\frac{21}{11}$ − 1 = $\frac{10}{11}$

Last term, l = $\frac{171}{11}$

a + (n − 1) d = $\frac{171}{11}$

1 + (n − 1) $\frac{10}{11}$ = $\frac{171}{11}$

$\frac{10n}{11} = \frac{171}{11} - 1 + \frac{10}{11}$

$\frac{10n}{11} = \frac{171-11+10}{11}$

10n = 170

n = 17

Hence, value of n is 17

23. Question

If an A.P. consists of n terms with first term a and nth term Is how that the sum of the m^{th} term from the beginning and the m^{th} term from the end is (a+l).

Answer

Given, a = a

Last term, l = l

We have to prove that the sum of the m^{th} term from the beginning and m^{th} term from the end is (a + 1)

Now, m^{th} term from the beginning,

$a_{m(b)} = a + (m - 1) l$

= a + ml – l (i)

Again m^{th} term from the end

$a_{m(e)} = l - (m - 1)l$

= l – ml + l = 2l – ml (ii)

By adding (i) and (ii), we get

a + ml – l + 2l – ml = a + l

Hence, proved

24. Question

Find the arithmetic progression whose third term is 16 and seventh term exceeds its fifth term by 12.

Answer

Given, $a_3 = 16$

a + 2d = 16 (i)

$a_5 = a + (5 - 1) d$

= a + 4d

$a_7 = a + (7 -1)d$

= a + 6d

Acc. To question,

$a_7 = 12 + a_5$ (ii)

Put the value of a_5 and a_7 in (ii), we get

a + 6d = 12 + a + 4d

2d = 12 + a − a

2d = 12

d = 6

From equation (i),

16 = a + 2(6)

16 = a + 12

a = 4

We have to find A.P.

$a_1 = 4$

$a_2 = a + d = 4 + 6 = 10$

$a_3 = a + 2d = 4 + 2(6) = 16$

$a_4 = a + 3d = 4 + 3(6) = 22$

Hence, the A.P. is 4 , 10, 16 , 22,...

25. Question

The 7^{th} term of an A.P. is 32 and its 13^{th} term is 62. Find the A.P.

Answer

$a_7 = 32$

a + 6d = 32 (i)

$a_{13} = a + 12d = 62$ (ii)

By subtracting (i) from (ii), we get

6d = 30

d= 5

From (i), a = 32 – 6(5) = 2

We have to find A.P.

$a_1 = 2$

$a_2 = a + d = 2 + 5 = 7$

$a_3 = a + 2d = 2 + 2(5) = 12$

$a_4 = a + 3d = 2 + 3(5) = 17$

Hence, A.P. is 2, 7, 12, 17,...

26. Question

Which term of the A.P. 3,10,17,...will be 84 more than its 13th term?

Answer

$a = 3, d = 7$

$a_{13} = 3 + (13 - 1)7$

$= 3 + 84 = 87$

Now, nth term is 84 more than the 13th term

$a_n = 84 + a_{13}$

$= 84 + 87$

$= 171$

Now we have to find n,

$a_n = a + (n - 1) d$

$171 = 3 + (n - 1) 7$

$7n = 175$

$n = 25$

Hence, 25th term of the given A,.P. is 84 more than its 13th term

27. Question

Two arithmetic progressions have the same common difference. The difference between their 100th terms is 100, what is the difference between their 1000th terms?

Answer

Two arithmetic progressions have the same common difference.

Let, common difference = d

First term of first A.P. = a

First term of second A.P. = a'

Given that difference between their 100th term is 100

100th term of first A.P., $a_{100} = a + 99d$

100th term of second A.P., $a'_{100} = a' + 99d$

Acc. To question,

$a_{100} - a'_{100} = 100$

$a + 99d - a' - 99d = 100$

$a - a' = 100$ (i)

1000^{th} term of first A.P., $a_{1000} = a + 999d$

1000^{th} term of second A.P., $a'_{1000} = a' + 999d$

Acc. To question,

$a_{1000} - a'_{1000} = a + 999d - a' - 999d$

$= a - a'$

$a_{1000} - a'_{1000} = 100$ [from (i)]

Hence, difference between 1000^{th} term of two A.P. is 100

28. Question

For what value of n, the nth terms of the arithmetic progressions $63, 65, 67, \ldots$ and $3, 10, 17, \ldots$ are equal?

Answer

Given, A.P. = 63, 65, 67,...

A.P.' = 3, 10, 17,...

Let for A.P.,

First term, a = 63, d = 65 − 63 = 2

And n^{th} term = a_n

$a_n = 63 + (n - 1) 2$

$= 63 + 2n - 2$

$= 61 + 2n$

For A.P.'

First term, a = 3, d = 10 − 3 = 7

n^{th} term = a_n

Now n^{th} term of both A.P. are equal

$61 + 2n = -4 + 7n$

$7n - 2n = 61 + 4$

$n = 13$

Hence, 13^{th} term of both the A.P. are equal

29. Question

How many multiples of 4 lie between 10 and 250?

Answer

Let, multiple of 4 lies between 10 and 250

12, 16, 20, 24,...248

We know, $a_n = a + (n - 1)d$

First term, a = 12

C0mmon difference, d = 16 − 12 = 4

Last term, $a_n = 248$

$a_n = a + (n - 1)d$

248 = 12 + (n − 1) 4

248 = 12 + 4n − 4

4n = 248 − 12 + 4

4n = 240

n = 60

Hence, multiple of 4 lies between 10 and 250 is 60

30. Question

How many three digit numbers are divisible by 7?

Answer

Let, three digit number divisible by 7 are

105, 112, 119,....994

Here, First term, a = 105

Common difference, d = 112 − 105 = 7

And last term, $a_n = 994$

$a_n = a + (n - 1) d$

994 = 105 + (n − 1) 7

994 = 105 + 7n − 7

994 = 98 + 7n

7n = 896

$n = 128$

Hence, three digit number that are divisible by 7 are 128

31. Question

Which term of the arithmetic progression 8, 14, 20, 26, ... will be72 more than its 41th term?

Answer

$a = 8$, $d = 6$,

Last term = a_n

$a_n = a + (n - 1) d$

$= 8 + (n - 1) 6$

$= 2 + 6n$ (i)

Let, 41st term, $a_{41} = 8 + (41 - 1) 6$

$= 8 + 40 * 6 = 248$

Now the term is 72 more than its 41st term

$a_n = 72 + a_{41}$

$= 72 + 248 = 320$

Putting this value in (i), we get

$320 = 2 + 6n$

$6n = 318$

$n = 53$

Hence, 53rd term of the given A.P. is 72 more than its 41st term

32. Question

Find the term of the arithmetic progression 9, 12, 15, 18, ... which is 39 more than its 36th term.

Answer

First term, $a = 9$, $d = 12 - 9 = 3$

Let, last term be a_n

$a_n = a + (n - 1) d$

$= 9 + (n - 1)3$

$= 9 + 3n - 3 = 6 + 3n$ (i)

Let, 36th term, $a_{36} = a + 35d$

$= 9 + 35 (3) = 114$

Now the term is 39 more than its 36th term

$a_n = 39 + a_{36}$

$= 39 + 114 = 153$

Putting the value in (i), we get

$153 = 6 + 3N$

$3n = 153 - 6$

$3n = 147$

$n = 49$

Hence, 49th term of the given A.P. is 39 more than its 36th term

33. Question

Find the 8th term from the end of the A.P. 7,10,13,...,184

Answer

$a = 7, d = 3$

Last term, $a_n = 184$

$a_n = a + (n - 1) d$

$184 = 7 + (n - 1) 3$

$184 = 7 + 3n - 3$

$180 = 3n$

$n = 60$

Now we have to find last 8th term, it means = 60th – 7 = 53rd term

$a_{53} = 7 + (53 - 1)3$

$= 7 + 52 * 3$

$= 7 + 156 = 163$

Hence, 8th term from the end of given is 163

34. Question

Find the 10th term from the end of the A.P. 8, 10, 12, ..., 126

Answer

Given,First term, a = 8Common difference, d = 2

Let the number of terms be 'n'We know, nth term of an AP is$a_n = a + (n - 1)d$where ‘a’ and ‘d’ are first term and common difference of AP respectivelyLast term, $a_n = 126$

⇒ $a_n = a + (n - 1) d$

⇒ $126 = 8 + (n - 1) 2$

⇒ $120 = 2n$

⇒ $n = 60$

Now we have to find last 10^{th} term, means = 60^{th} term – 10 + 1 = 51^{th} term

Now, $a_{51} = 8 + (51 - 1)2$

$= 8 + 50(2) = 8 + 100 = 108$

Hence, 10^{th} term from the last of given A.P. is 108

35. Question

The sum of 4^{th} and 8^{th} terms of an A.P. is 24 and the sum of 6^{th} and 10^{th} terms is 44. Find the A.P.

Answer

Given, $a_4 + a_8 = 24$ (i)

$a_6 + a_{10} = 44$ (ii)

We know, $a_n = a + (n - 1) d$

4^{th} term, $a_4 = a + 3d$

6^{th} term, $a_6 = a + 5d$

8^{th} term, $a_8 = a + 7d$

10^{th} term, $a_{10} = a + 9d$

Putting the value of a_4 and a_8 in (i), we get

$a + 3d + a + 7d = 24$

$2a + 10d = 24$ (iii)

Put the value of a_6 and a_{10} in (ii), we get

$a + 5d + a + 9d = 44$

$2a + 14d = 44$ (iv)

By subtracting (iii) from (iv), we get

$4d = 20$

$d = 5$

Now putting value of d in (iii), we get

$2a = 24 - 10(5)$

$a = -13$

$a_1 = -13$

$a_2 = a + d = -13 + 5 = - 8$

$a_3 = a + 2d = -13 + 10 = -3$

Hence, the A.P. is -13, -8, -3,...

36. Question

Which term of the A.P. 3,15,27,39,... will be 120 more than its 21st term?

Answer

$a = 3$, $d = 15 - 3 = 12$

Let last term be a_n

$a_n = a + (n - 1) d$

$= 3 + (n - 1) 12$

$= 12n - 9$ (i)

$a_{21} = a + 20d$

$= 3 + 20(12) = 243$

Now, the term is 120 more than the 21st term

$a_n = 120 + a_{21}$

$= 120 + 243$

$= 363$

Putting this value in (i), we get

$363 = 12n - 9$

$12n = 363 + 9$

$n = 31$

Hence, 31st term of given A.P. is 120 more than its 21st term

37. Question

The 17th term of an A.P. is 5 more than twice its 8th term. If the 11th term of the A.P. is 43, find the n^{th} term.

Answer

Given: The 17th term of an A.P. is 5 more than twice its 8th term and the 11th term of the A.P. is 43.

To find: the n^{th} term.

Solution: Given, $a_{17} = 5 + 2(a_8)$ ......(i)

And$a_{11} = 43$As $a_n = a + (n-1)\, d$ For n=11,

$a + 10d = 43a = 43 - 10d$ ......(ii)For n = 8,

$a_8 = a + 7d$For n=17,

$a_{17} = a + 16d$Put the value of a from (ii)

$\Rightarrow a_{17} = 43 - 10d + 16d$

$\Rightarrow a_{17} = 43 + 6d$

Putting the value of a_8 and a_{17} in (i), we get

$\Rightarrow 43 + 6d = 5 + 2(a + 7d) \Rightarrow 43 + 6d = 5 + 2a + 14d$

$\Rightarrow 43 - 5 = 2a + 14d - 6d$

$\Rightarrow 38 = 2a + 8d$

$\Rightarrow 38 = 2(43 - 10d) + 8d$ [from (ii)]

$\Rightarrow 38 = 86 - 20d + 8d$

$\Rightarrow 38 = 86 - 12d$

$\Rightarrow 12d = 86 - 38 \Rightarrow 12d = 48$

$\Rightarrow d = 4$

From (ii), $a = 43 - 10d$

$\Rightarrow 43 - (10 \times 4) = 43 - 40 = 3$

We know, n^{th} term of A.P., $a_n = a + (n - 1)\, d$

$a_n = 3 + (n - 1)\, 4$

$a_n = 3 + 4n - 4$ $a_n = 4n - 1$

Hence, n^{th} term is $4n - 1$.

38. Question

Find the number of all three digit natural numbers which are divisible by 9.

Answer

Let the three digit numbers divisible by 9 are

108, 117, 126,… ,999

a=108, d=9, last term, l= 999

Number of terms, n $= \frac{l-a}{d} + 1$

$=\frac{999-108}{9}+1$

$=\frac{891}{9}+1$

$=100$

Hence, the number of all three digit natural numbers which are divisible by 9 are 100

39. Question

The 19th term of an A.P. is equal to three times its sixth term. If its 9th term is 19, find the A.P.

Answer

Given: $a_9 = 19$

$a + 8d = 19$ (i)

Acc. to question,

$a_{19} = 3a_6$ (ii)

$a_{19} = a + 18d$

$a_6 = a + 5d$

Putting the value of a_{19} and a_6 in (ii)

$a + 18d = 3(a + 5d)$

$3d = 2a$

$3d = 2(19 - 8d)$ (from (i))

$19d = 38$

$d = 2$

Now, putting the value of d in (i)

$a = 19 - 8(2) = 3$

$a_1 = a = 3$

$a_2 = a + d = 3 + 2 = 5$

$a_3 = a + 2d = 3 + 2(2) = 7$

Hence, the A.P. is 3, 5, 7, 9, ...

40. Question

The 9th term of an A.P. is equal to 6 times its second term. If its 5th term is 22, find the A.P.

Answer

Given: $a_5 = 22$

a + 4d=22 (i)

Acc. to question,

a_9 = $6a_2$ (ii)

a_9= a +8d

a_2=a+d

Putting the value of a_9 and a_2 in (ii)

a +8d= 6(a +d)

2d=5a

2d=5(22 -4d)

22d=110

d=5

Now, putting the value of d in (i)

a=22 -4(5)=2

a_1 = a=2

a_2 =a +d=2 +5=7

a_3 =a +2d=2 + 2(5)=12

Hence, the A.P. is 2,7,12,17,...

41. Question

The 24th term of an A.P. is twice its 10th term. Show that its 72nd term is 4 times its 15th term.

Answer

Given: a_{24}=$2a_{10}$

a +23d= 2(a +9d)

5d=a (i)

To Prove:a_{72} =$4a_{15}$

Proof: L.H.S. =a_{72}

= a +71d

=5d +71d [from (i)]

=76d

R.H.S= $4a_{15}$

=4(a +14d)

=4a +56d

=4(5d) +56d [from (i)]

=20d +56d

=76d

Since, L.H.S=R.H.S.

Hence proved

42. Question

Find the number of natural numbers between 101 and 999 which are divisible by both 2 and 5.

Answer

We have to find the numbers between 101 and 999 which are divisible by both 2 and 5

That means the numbers divisible by 10 between 101 and 999.

Let, the required divisible numbers be,

110, 120, 130, …, 990

a=110, d=10, last term, l=990

Number of terms, n $=\frac{l-a}{d}+1$

$=\frac{990-110}{10}+1$

$=\frac{880}{10}+1$

=89

Hence, the number of natural numbers between 101 and 999 which are divisible by both 2 and 5 are 89.

43. Question

If the seventh term of an A.P. is 1/9 and its ninth term is 1/7, find its $(63)^{rd}$ term.

Answer

We know, nth term of an AP is $a_n = a + (n - 1)d$ where 'a' and 'd' are first term and common difference of AP respectively

Given,

7th term of AP $= \frac{1}{9}$

$$\Rightarrow a_7 = \frac{1}{9}$$

$$\Rightarrow a + 6d = \frac{1}{9} \quad [1]$$

9th term of AP $= \frac{1}{7}$

$$\Rightarrow a_9 = \frac{1}{9}$$

$$\Rightarrow a + 8d = \frac{1}{7} \quad [2]$$

Subtracting equation [1] from [2], we get

$$\Rightarrow a + 8d - a - 6d = \frac{1}{7} - \frac{1}{9}$$

$$\Rightarrow 2d = \frac{2}{63}$$

$$\Rightarrow d = \frac{1}{63}$$

Putting this value of d in equation [1], we get

$$\Rightarrow a + 6\left(\frac{1}{63}\right) = \frac{1}{9}$$

$$\Rightarrow a + \frac{2}{21} = \frac{1}{9}$$

$$\Rightarrow a = \frac{1}{9} - \frac{2}{21}$$

$$\Rightarrow a = \frac{1}{63}$$

Now, 63rd term = a + 62d

$$= \frac{1}{63} + 62\left(\frac{1}{63}\right)$$

$$= 1$$

Hence Proved!

44. Question

The sum of 5^{th} and 9^{th} terms of an AP is 30. If its 25^{th} term is three times its 8^{th} term, find the AP.

Answer

$a_5 + a_9 = 30$

(a +4d) + (a +8d)=30

2a +12d=30

a + 6d=15 (i)

Acc. To question,

$a_{25} = 3a_8$

a+ 24d = 3(a +7d)

3d=2a

3d=2(15 -6d) [from(i)]

15d=30

d=2

Putting the value of d in (i),

a= 15 – 6(2)=3

a_1 = a=3

a_2 =a +d=3 +2=5

a_3 =a +2d=3 + 2(2)=7

Hence, the A.P. is 3,5,7,9,...

45. Question

Find where 0 (zero) is a term of the A.P. 40,37,34,31,...

Answer

a=40, d=37 -40= -3

Let, a_n =0

a + (n -1) d =0

40 + (n -1) (-3) =0

-3n = -43

n=43/3

Since, 'n' can't be a fraction

Hence, the answer is no

46. Question

Find the middle term of the A.P. $213, 205, 197, \ldots, 37$.

Answer

a=213, l=37

Middle term of A.P. $= \frac{1}{2}(a + l)$

$= \frac{1}{2}(213 + 37)$

$= \frac{250}{2}$

=125

47. Question

If the 5th term of an A.P. is 31 and 25th term is 140 more than the 5th term, find the A.P.

Answer

$a_5=31$

a +4d =31 (i)

$a_{25}= 140 + a_5$

a + 24d= 140 + 31

31 -4d +24d=171 [from (i)]

20d=140

d=7

Putting the value of d in (i)

a= 31 – 4(7) = 3

a_1 = a=3

a_2 =a +d=3 +7=10

a_3 =a +2d=3 + 2(7) =17

Hence, the A.P. is 3, 10, 17,...

Exercise 9.4

1. Question

The sum of three terms of an A.P. is 21 and the product of the first and the third terms exceeds these term by 6, find three terms.

Answer

Let the three terms be a - d, a, a + d

a − d + a + a − d = 21 (given)

3a = 21

a = 7

According to question,

(a − d) (a + d) = a + 6

$a^2 - d^2 = a + 6$

$7^2 - d^2 = 7 + 6$

$49 - d^2 = 13$

$d^2 = 36$

d = 6

Therefore, a − d = 7 − 6 = 1

a + d = 7 + 6 = 13

Thus, the three terms are 1, 7 and 13.

2. Question

Three numbers are in A.P. If the sum of these numbers be 27 and the product 648, find the numbers.

Answer

Let the three numbers be a − d, a, a + d

a − d + a + a + d = 27

3a = 27

a = 9

According to question,

(a − d) (a) (a + d) = 648

$(a^2 - d^2)(a) = 648$

$(9^2 - d^2)\ 9 = 648$

$(81 - d^2) = 72$

$d^2 = 9$

d = 3

Therefore, a − d = 9 − 3 = 6

A + d = 9 + 3 = 12

3. Question

Find the four numbers in A.P. whose sum is 50 and in which the greatest number is 4 times the least.

Answer

Let the four numbers be (a − 3d), (a − d), (a + d) and (a + 3d)

a − 3d + a − d + a + d + a + 3d = 50

4a = 50

$a = \frac{25}{2}$

According to question,

(a + 3d) = 4(a − 3d)

a + 3d = 4a − 12d

3a = 15d

$5d = \frac{25}{2}$

$d = \frac{5}{2}$

Therefore,

$a - 3d = \frac{25}{2} - \frac{15}{2} = 5$

$a - d = \frac{25}{2} - \frac{5}{2} = 10$

$a + d = \frac{25}{2} + \frac{5}{2} = 15$

$a + 3d = \frac{25}{2} + \frac{15}{2} = 20$

4. Question

The angles of a quadrilateral are in A.P. whose common difference is 10°. Find the angles.

Answer

Let the angles be (a − 3d), (a − d), (a + d) and (a + 3d)

a − 3d + a − d + a + d + a + 3d = 360^o (sum of angles of quadrilateral)

4a = 360^o

a = 90^o

According to question,

$(a + d) - (a - d) = 10^o$

$2d = 10^o$

$d = 5^o$

Therefore, $(a - 3d) = 90^o - 15^o = 75^o$

$(a - d) = 90^o - 5^o = 85^o$

$(a + d) = 90^o + 5^o = 95^o$

$(a + 3d) = 90^o + 15^o = 105^o$

5. Question

The sum of three numbers in A.P. is 12 and the sum of their cube is 288. Find the numbers.

Answer

Let the numbers be $(a - d)$, a, $(a + d)$

$A - d + a + a + d = 12$

$3a = 12$

$a = 4$

According to question,

$(a - d)^3 + a^3 + (a + d)^3 = 288$

$a^3 - d^3 - 3ad(a - d) + a^3 + a^3 + d^3 + 3ad(a + d) = 288$

$3a^3 - 3a^2d + 3ad^2 + 3a^2d + 3ad^2 = 288$

$3(4)^3 + 6(4)d^2 = 288$

$24d^2 = 96$

$d^2 = 4$

d = �4

Therefore, when $a = 4$ and $d = 2$

$a - d = 4 - 2 = 2$

$a + d = 4 + 2 = 6$

When $a = 4$ and $d = -2$

$a - d = 4 + 2 = 6$

$a + d = 4 - 2 = 2$

6. Question

Find the value of x for which (8x+4), (6x-2), (2x+7) are in A.P.

Answer

We know that, if three numbers are a,b,c are in AP then, 2b = a + c

According to question, (8x+4), (6x-2), (2x+7) are in AP.

Hence,

$2(6x - 2) = 8x + 4 + 2x + 7$

$12x - 4 = 10x + 11$

$2x = 15$

$x = 15/2 = 7.5$

7. Question

If x+1, 3x and 4x+2 are in A.P, find the value of x.

Answer

Given terms: x+1, 3x and 4x+2

Since, the given terms are in A.P.

Therefore, their common difference will be same

$3x - (x + 1) = (4x + 2) - 3x$

$3x - x - 1 = 4x + 2 - 3x$

$2x - 1 = x + 2$

$x = 3$

8. Question

Show that $(a-b)^2$, (a^2+b^2) and $(a + b)^2$ are in A.P.

Answer

The terms given below are : $(a-b)^2$, (a^2+b^2) and $(a + b)^2$

Common difference, $d_1 = a^2 + b^2 - (a - b)^2$

$d_1 = a^2 + b^2 - (a^2 + b^2 - 2ab)$

$d_1 = a^2 + b^2 - a^2 - b^2 + 2ab$

$\mathbf{d_1 = 2ab}$

Common difference, $d_2 = (a + b)^2 - (a^2 + b^2)$

$d_2 = a^2 + b^2 + 2ab - a^2 - b^2$

d_2 = 2ab

Since, $d_1 = d_2$ i.e. the common difference is same.

Therefore, the given terms are in A.P.

Exercise 9.5

1. Question

Find the sum of the following arithmetic progressions:

(i) $50, 46, 42, \ldots$ to 10 terms

(ii) $1, 3, 5, 7, \ldots$ to 12 terms

(iii) $3, 9/2, 6, 15/2, \ldots$ to 25 terms

(iv) $41, 36, 31, \ldots$ to 12 terms

(v) $a+b, a-b, a-3b, \ldots$ to 22 terms

(vi) $(x-y)^2, (x^2+y^2), (x+y)^2, \ldots,$ to n terms

(vii) $\frac{x-y}{x+y}, \frac{3x-2y}{x+y}, \frac{5x-3y}{x+y}, \ldots$ to n terms

(viii) $-26, -24, -22, \ldots$ to 36 terms

Answer

(i) $50, 46, 42, \ldots$ to 10 terms

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{10} = \frac{10}{2}[100 + 9(-4)]$

$= 5[100 - 36]$

$= 5(64) = 320$

(ii) $1, 3, 5, 7, \ldots$ to 12 terms

$S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{12}{2}[2 + (12-1)2]$

$= 6(2 + 22)$

$= 144$

(iii) $3, 9/2, 6, 15/2, \ldots$ to 25 terms

$S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{25}{2}\ [2\ (3) + 24(\frac{3}{2})]$

$= \frac{25}{2}\ [6 + 36]$

= 525

(iv) $41, 36, 31, \ldots$ to 12 terms

$S_n = \frac{n}{2}\ [2a + (n - 1)\ d]$

$= \frac{12}{2}\ [82 - 55]$

= 6 [27] = 162

(v) $a + b, a - b, a - 3b, \ldots$ to 22 terms

$S_n = \frac{n}{2}\ [2a + (n - 1)\ d]$

$= \frac{22}{2}\ [2(a + b) + (21)\ (-2b)]$

= 11 [2a + 2b − 42b]

= 11 [2a − 40b]

= 22a − 440b

(vi) $(x - y)^2, (x^2 + y^2), (x + y)^2, \ldots,$ to n terms

$S_n = \frac{n}{2}\ [2a + (n - 1)\ d]$

$= \frac{n}{2}\ [2(x - y)^2 + (n - 1)\ (2xy)]$

$= \frac{n}{2}\ (2)\ [(x - y)^2 + (n - 1)\ xy]$

$= n\ [(x - y)^2 + (n - 1)\ xy]$

(vii) $\frac{x - y}{x + y}, \frac{3x - 2y}{x + y}, \frac{5x - 3y}{x + y}, \ldots$ to n terms

$S_n = \frac{n}{2}\ [2a + (n - 1)\ d]$

$= \frac{n}{2}\ [2(\frac{x-y}{x+y}) + (n - 1)\ xy]$

$= \frac{n}{2(x+y)}\ \{2(x - y) + (n - 1)\ (2x - y)\}$

$= \frac{n}{2(x+y)}\ \{n\ (2x - y) - y\}$

(viii) $-26, -24, -22, \ldots$ to 36 terms

$S_n = \frac{n}{2}\ [2a + (n - 1)\ d]$

$= \frac{36}{2}$ [-52 + (35) 2]

= 18 [-52 + 70]

= 18 [18] = 324

2. Question

Find the sum on n term of the A.P. $5, 2, -1, -4, -7, \ldots,$

Answer

$S_n = \frac{n}{2}$ [2a + (n – 1) d]

$= \frac{n}{2}$ [10 + (n – 1) -3]

$= \frac{n}{2}$ [13 – 3n]

3. Question

Find the sum of n terms of an A.P. whose n^{th} term is given by $a_n = 5 - 6n$

Answer

Put n = 1

a_1 = 5 – 6(1) = 5 – 6 = -1

Put n = 2

a_2= 5 – 6(2) = 5 – 12 = -7

a = -1, d = -7 + 1 = -6

$S_n = \frac{n}{2}$ [2a + (n – 1) d]

$= \frac{n}{2}$ [2(-1) + (n -1) (-6)]

$= \frac{n}{2}$ [-1 + 3n + 3]

= n [2 -3n]

4. Question

If the n sum of a certain number of terms starting from first term of an A.P. is $25, 22, 19, \ldots,$ is 116. Find the last term.

Answer

a = 25, d = -3

$S_n = \frac{n}{2}$ [2a + (n – 1) d]

116 $= \frac{n}{2}$ [50 – 3n +3]

$116 = \frac{n}{2}[53 - 3n]$

$232 = 53n - 3n^2$

$3n^2 - 53n + 232 = 0$

$n = 8$ or $n = \frac{29}{3}$, which isn't possible as n must be a natural number.

Therefore, n = 8

$\frac{n}{2}[a + l] = 116$

$\frac{8}{2}[25 + l] = 116$

$25 + l = 29$

$l = 4$

Hence, the last term is 4

5 A. Question

How many terms of the sequence 18,16,14,...should be taken so that their sum is zero?

Answer

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$0 = \frac{n}{2}[2(18) + (n - 1)(-2)]$

$38n - 2n^2 = 0$

n = 0 (which is not possible)

n = 19

Therefore, n = 19 terms

5 B. Question

How many terms are there in the A.P. whose first and fifth terms are 14 and 2 respectively and the sum of the terms is 40?

Answer

$a_5 = a + (n - 1)d$

$2 = -14 + 4d$

$16 = 4d$

$d = 4$

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$40 = \frac{n}{2}$ [2(-14) + (n -1) (4)]

40 = 12n^2 – 16n

2n^2 – 16n -40 = 0

n^2 – 8n -20 = 0

n^2 – 10n + 2n – 20 = 0

(n – 10) (n + 2) = 0

n = 10 and n = -2 (not possible)

5 C. Question

How many terms of the A.P. $9, 17, 25, \ldots$ must be taken so that their sum is 636?

Answer

$S_n = \frac{n}{2}$ [2a + (n – 1) d]

$636 = \frac{n}{2}$ [2(9) + (n – 1) 8]

636 = n [9 + 4n -4]

636 = 5n + 4n^2

4n^2 + 5n – 636 = 0

n = 12 or n = $\frac{-53}{4}$ (not possible)

5 D. Question

How many terms of the A.P. $63, 60, 57, \ldots$ must be taken so that their sum is 693?

Answer

$S_n = \frac{n}{2}$ [2a + (n – 1) d]

$693 = \frac{n}{2}$ [2(63) + (n – 1) (-3)]

$693 = \frac{n}{2}$ [126 -3n + 3]

462 = n [43 – n]

n^2 – 43n + 462 = 0

n = 22 or n = 21

6. Question

The first and the last terms of an A.P. are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?

Answer

a = 17, l = 350, d = 9

Number of terms, n = $\frac{l-a}{d}$ + 1

= $\frac{350-17}{9}$ + 1

= $\frac{333+9}{9}$ = $\frac{342}{9}$ = 38

S_n= $\frac{n}{2}$ (a + l)

= $\frac{38}{2}$ (350 + 17)

= 19 (367) = 6973

7. Question

The third term of an A.P. is 7 and the seventh term exceeds three times the third term by 2. Find the first term, the common difference and the sum of first 20 terms.

Answer

We know, nth term of an AP isa_n = a + (n − 1)dwhere ‘a’ and ‘d’ are first term and common difference of AP respectivelyGiven, third term of an A.P. is 7a_3 = 7⇒ a + 2d = 7   (i)the seventh term exceeds three times the third term by 2

⇒ a_7 = 3a_3 + 2⇒ a + 6d = 3(7) + 2⇒ 7 − 2d +6d = 21 + 2⇒ 4d = 16⇒ d = 4

From (i), a = 7 − 2(4)

= -1

Also, we know sum of first 'n' terms of an AP is $S_n = \frac{n}{2}[2a + (n-1)d]$ Sum of first 20 terms is

$\Rightarrow S_{20} = \frac{20}{2}[2(-1) + 19(4)]$ = 10[-2 + 76]= 740

8. Question

The first term of an A.P. is 2 and the last term is 50. The sum of all these terms is 442. Find the common difference.

Answer

a = 2, l = 50

S_n= $\frac{n}{2}$ [a + l]

442 = $\frac{n}{2}$ [50 +2]

n = $\frac{884}{52}$ = 17

$n = \frac{l-a}{d} + 1$

$17 = \frac{50-2}{d} + 1$

16d = 48

d = 3

9. Question

If 12th term of an A.P. is 13 and the sum of the first four terms is 24, what is the sum of first 10 terms?

Answer

$a_{12} = -13$

a + (12 – 1) d = -13

a + 11d = -13

$S_4 = \frac{4}{2}$ [2a + (4 – 1) d]

24 = 2 [2a + 3d]

12 = 2a + 3d

12 = 2 (-11d – 13) + 3d

12 = -22d – 26 + 3d

38 = -19d

d = -2

a = -13 + 11(2)

= -13 + 22

= 9

$S_{10} = \frac{10}{2}$ [2(9) + (10 – 1) (-2)]

= 5 [18 – 18]

= 0

10. Question

Find the sum of first 22 terms of an A.P. in which d = 22 and a_{22} = 149.

Answer

$a_{22} = a + 21d$

149 = a + 21(22)

a = 149 – 462

a = -313

$S_{22} = \frac{22}{2}$ [2a + (22 – 1) d]

= 11 [-626 + 462]

= 11 (-164)

= -1804

11. Question

Find the sum of all natural numbers between a and 100, which are divisible by 3.

Answer

a = 3, l = 99, n = 33

$S_{33} = \frac{33}{2}$ (a + l)

= $\frac{33}{2}$ (3 + 99)

= $\frac{33}{2}$ (102)

= 33 (51) = 1683

12. Question

Find the sum of first n odd natural numbers.

Answer

a = 1, d = 2

Sum of first n odd numbers, $S_n = \frac{n}{2}$ [2a + (n – 1) d]

= $\frac{n}{2}$ [2(1) + (n – 1) 2]

= n [1 + n – 1]

= n^2

13. Question

Find the sum of all odd numbers between

(i) 0 and 50

(ii) 100 and 200

Answer

(i) 0 and 50

a = 1, l = 49, n = 25

$S_{25} = \frac{25}{2}[a + l]$

$= \frac{25}{2}[1 + 49]$

$= \frac{25}{2}(50) = 625$

(ii) 100 and 200

a = 101, l = 199, n = 50

$S_{50} = \frac{50}{2}[a + l]$

= 25 (101 + 199)

= 25 (300) = 7500

14. Question

Show that the sum of all odd integers between 1 and 1000 which are divisible by 3 is 83667.

Answer

a = 3, l = 999, n = $\frac{333+1}{2}$ = 167

$S_n = \frac{167}{2}(a + l)$

$= \frac{167}{2}(3 + 999)$

$= \frac{167}{2}(1002)$

= 167 (501) = 83667

15. Question

Find the sum of all integers between 84 and 719, which are multiples of 5.

Answer

To find: the sum of all integers between 84 and 719, which are multiples of 5.

Solution: The smallest and largest digits between 84 and 719 which are divisible by 5 is 85 and 715.So the sequence will be 85,90,........... 715. a = 85, d = 5, l = 715From the formula l=a+(n-1)dThe number of terms are:

n = $\frac{l-a}{d}$ + 1

n= $\frac{715-85}{5}$ + 1

n= $\frac{635}{5}$ n= 127

So there are 127 terms between 84 and 719.Now to calculate the sum of 127 terms use the formula:

$$s_n = \frac{n}{2}(a + l)$$

$\Rightarrow S_{127} = \frac{127}{2}(a + l)$

$\Rightarrow S_{127} = \frac{127}{2}(85 + 715)$

$\Rightarrow S_{127} = \frac{127}{2}(800) = 50800$So the sum of the terms between 84 and 719 is 50800.

16. Question

Find the sum of all integers between 50 and 500, which are multiples of 7.

Answer

a = 56, d = 7, l = 497

$n = \frac{l-a}{d} + 1$

$= \frac{497-56}{7} + 1$

$= \frac{441}{7} + 1 = \frac{448}{7}$

= 64

$S_{64} = \frac{64}{2}(a + l)$

= 32 (56 + 497)

= 32 (553) = 17696

17. Question

Find the sum of all even integers between 101 and 999.

Answer

a = 102, l = 998

$n = \frac{999-101}{2} = 449$

$S_{449} = \frac{449}{2}(a + l)$

$= \frac{449}{2}(102 + 998)$

$= \frac{449(1100)}{2} = 246950$

18. Question

Find the sum of all integers between 100 and 550, which are multiples of 9.

Answer

a = 108

L = 549

D = 9

549 = 108 + (n - 1)9

549 = 99 + 9n

9n = 450

n = 50

$S_n = \frac{50}{2}$ [216 + 49 (9)]

= 16425

19. Question

In an A.P. if the first term is 22, the common difference is – 4 and the sum to n terms is 64, find n.

Answer

a = 22, d = -4

S_n = 64

$64 = \frac{n}{2}(2a + (n - 1) d)$

n [44 – (n – 1)4] = 128

n (44 – 4n + 4) = 128

$48n - 4n^2 = 128$

$n^2 - 12n + 32 = 0$

$n^2 - 8n - 4n + 32 = 0$

(n – 8) (n – 4) = 0

n = 8 or n = 4

20. Question

In an A.P. If the 5^{th} and 12^{th} terms are 30 and 65 respectively, what is the sum of first 20 terms?

Answer

According to question,

$a_5 = 30$

a + 4d = 30

$a = 30 - 4d$ (i)

$a_{12} = 65$

$a + 11d = 65$

$30 - 4d + 11d = 65$ [from (i)]

$7d = 35$

$d = 5$

Put d in (i)

$a = 30 - 4(5)$

$= 10$

$S_{20} = \frac{20}{2}[2(a) + (20 - 1)d]$

$= 10 [2(10) + 19(5)]$

$= 10 \{20 + 95\}$

$= 1150$

21. Question

Find the sum of the first

(i) 11 terms of the A.P : $2, 6, 10, 14, \ldots$

(ii) 13 terms of the A.P : $-6, 0, 6, 12, \ldots$

(iii) 51 terms of the A.P. whose second term is 2 and fourth term is 8.

Answer

(i) 11 terms of the A.P : $2, 6, 10, 14, \ldots$

$a = 2, d = 6 - 2 = 4$

$S_{11} = \frac{11}{2}[2(a) + 10d]$

$= \frac{11}{2}[2(2) + 10(4)]$

$= 11 [2 + 20]$

$= 242$

(ii) 13 terms of the A.P : $-6, 0, 6, 12, \ldots$

$a = -6, d = 0 + 6 = 6$

$S_{13} = \frac{13}{2}[2(a) + 12d]$

$= 13 [-6 + 6(6)]$

= 13 [-6 + 36]

= 13 (30) = 390

(iii) 51 terms of the A.P. whose second term is 2 and fourth term is 8.

$a_2 = 2$

a + d = 2 (i)

$a_4 = 8$

a + 3d = 8

2 – d + 3d = 8

2 + 2d = 8

d = 3

a = -1

$S_{51} = \frac{51}{2}$ [2(a) + 50d]

= 51 [-1 + 25(3)]

= 51 (74)

= 3774

22. Question

Find the sum of

(i) The first 15 multiples of 8

(ii) The first 40 positive integers divisible by (a) 3 (b) 5 (c) 6.

(iii) All 3 – digit natural numbers which are divisible by 13.

(iv) All 3 – digit natural numbers, which are multiples of 11.

Answer

(i) The first 15 multiples of 8

a = 8, d = 8

$S_{15} = \frac{15}{2}$ [2(8) + 14(8)]

= 15[8 + 56]= 15[64]

= 960

(ii) The first 40 positive integers divisible by (a) 3 (b) 5 (c) 6.

(a) a = 3, d = 3

$S_{40} = \frac{40}{2}$ [2(a) + 39(d)]

= 20 [2(3) + 39(3)]

= 60 (41) = 2460

(b) a = 6, d = 5

$S_{40} = \frac{40}{2}$ [2(5) + 39(5)]

= 100 [41]

= 4100

(iii) All 3 – digit natural numbers which are divisible by 13.

a = 6, d = 6

$S_{40} = \frac{40}{2}$ [2(6) + 39(6)]

= 120 (41) = 4920

(iv) All 3 – digit natural numbers, which are multiples of 11.

a = 110, d = 11, l = 990

n = $\frac{l-a}{d}$ + 1

= $\frac{990-110}{11}$ + 1

= $\frac{880+11}{11} = \frac{891}{11}$

= 81

$S_{81} = \frac{81}{2}$ [2(110) + 80(11)]

= 8910 [1 + 4]

= 44550

23. Question

Find the sum :

(i) $2 + 4 + 6 + \ldots + 200$

(ii) $3 + 11 + 19 + \ldots + 803$

(iii) $(-5) + (-8) + (-11) + \ldots + (-230)$

(iv) $1 + 3 + 5 + 7 + \ldots + 199$

(v) $7 + 10\frac{1}{2} + 14 + \ldots + 84$

(vi) $34 + 32 + 30 + \ldots + 10$

(vii) $25 + 28 + 31 + \ldots + 100$

(viii) $18 + 15\frac{1}{2} + 13 + \ldots + \left(-49\frac{1}{2}\right)$

Answer

(i) $2 + 4 + 6 + \ldots + 200$

a = 2, d = 4 − 2 = 2, l = 200

n = $\frac{l-a}{d}$ + 1

= $\frac{200-2}{2}$ + 1 = 100

S_{100}= $\frac{100}{2}$ [2(2) + 99(2)]

= 100 [101] = 10100

(ii) $3 + 11 + 19 + \ldots + 803$

a = 3, d = 11 − 3 = 8, l = 803

S_n= $\frac{l-a}{d}$ + 1 = $\frac{803-3}{8}$ + 1

= 101

S_{101}= $\frac{101}{2}$ [2(3) + 100(8)]

= 101 (403) = 40703

(iii) $(-5) + (-8) + (-11) + \ldots + (-230)$

a = -5, d = -8 + 5 = -3, l = -230

n = $\frac{-230+5}{-3}$ + 1

= $\frac{228}{3}$ = 76

S_n= 38(5) [-2 − 45]

= 190 (-47) = -8930

(iv) $1 + 3 + 5 + 7 + \ldots + 199$

a = 1, d = 3 − 1 = 2, l = 199

n = $\frac{199-1}{2}$ + 1

= 100

$S_{100} = \frac{100}{2}[2(1) + 99(2)]$

$= 100\ (100) = 10000$

(v) $7 + 10\frac{1}{2} + 14 + ... + 84$

$a = 7,\ d = \frac{21}{2} - 7 = \frac{7}{2}$

Last term, $a_n = 84$

$a_n = a + (n - 1)d$

$84 = 7 + (n - 1)\frac{7}{2}$

$84 = \frac{14+7n-7}{2}$

$84 * 2 = 7 + 7n$

$168 = 7 + 7n$

$7n = 168 - 7$

$7n = 161$

$n = 23$

Sum of n^{th} term, $S_n = \frac{n}{2}[2a + (n - 1)d]$

$S_{23} = \frac{23}{2}[2(7) + (23 - 1)\frac{7}{2}]$

$= \frac{23}{2}[14 + 22 * \frac{7}{2}]$

$= \frac{23}{2}[14 + 77]$

$= \frac{23}{2} * 91 = \frac{2093}{2}$

Hence, sum of given A.P. is $\frac{2093}{2}$

(vi) $34 + 32 + 30 + ... + 10$

$a = 34,\ d = 32 - 34 = -2$

Last term, $a_n = 10$

$a_n = a + (n - 1)d$

$10 = 34 + (n - 1)\ (-2)$

$10 = 34 - 2n + 2$

$2n = 34 - 10 + 2$

2n = 26

n = 13

Sum of n terms,

$S_n = \frac{n}{2}$ [2a + (n – 1) d]

$S_{13} = \frac{13}{2}$ [2(34) + (13 – 1) (-2)]

= $\frac{13}{2}$ [68 + 12(-2)]

= $\frac{13}{2}$ [68 – 24]

= $\frac{13}{2}$ * 44 = 286

Hence, Sum of given A.P. is 286

(vii) $25 + 28 + 31 + ... + 100$

a = 25, d = 3

Last term, a_n = 100

A_n = a + (n -1) d

100 = 25 (n – 1)3

75 = 3n – 3

78 = 3n

n = 26

Sum of n terms, $S_n = \frac{n}{2}$ [2a + (n – 1) d]

= $\frac{26}{2}$ [2(25) + (26 – 1) 3]

= 13 [50 + 25 * 3]

= 13 * 125 = 1625

Hence, Sum of given A.P. is 1625

(viii) $18 + 15\frac{1}{2} + 13 + ... + \left(-49\frac{1}{2}\right)$

a = 18, d = $\frac{31}{2}$ - 18 = $\frac{-5}{2}$

Last term, $a_n = \frac{-99}{2}$

a_n = a+ (n – 1)d

$\frac{-99}{2}$ = 18 + (n – 1) ($\frac{-5}{2}$)

-99 = 36 + 5 – 5n

-140 = -5n

n = 28

S_{28}= $\frac{28}{2}$ [a + l]

= 14 (18 - $\frac{99}{2}$)

= 7 (36 – 99)

= 7 (-63) = -441

24. Question

Find the sum of the first 15 terms of each of the following sequences having n^{th} term as

(i) $a_n = 3 + 4n$

(ii) $b_n = 5 + 2n$

(iii) $x_n = 6 - n$

(iv) $y_n = 9 - 5n$

Answer

(i) $a_n = 3 + 4n$

Put n = 1

a_1 = 3 + 4(1) = 7

Put n = 15

a_{15} = 3 + 4(15) = 63 = l

Sum of 15 terms, S_{15} = $\frac{15}{2}$ [a + l]

= $\frac{15}{2}$ [7 + 63] = 525

(ii) $b_n = 5 + 2n$

Put n = 1

b_1 = 5 + 2(1) = 7

Put n = 15

b_{15} = 5 + 2(15) = 35 = l

Sum of 15 terms, $S_{15} = \frac{15}{2}[a + l]$

$= \frac{15}{2}[7 + 35] = 315$

(iii) $x_n = 6 - n$

Put n = 1

$x_1 = 6 - 1 = 5$

Put n = 15

$x_{15} = 6 - 15 = -9$

Sum of 15 terms, $S_{15} = \frac{15}{2}[a + l]$

$= \frac{15}{2}[5 - 9] = -30$

(iv) $y_n = 9 - 5n$

Put n = 1

$y_1 = 9 - 5(1) = 4$

Put n = 15

$y_{15} = 9 - 5(15) = -66$

Sum of 15 terms, $S_{15} = \frac{15}{2}[a + l]$

$= \frac{15}{2}[4 - 66] = -465$

25. Question

Find the sum of first 20 terms of these sequence whose n^{th} term is $a_n = An + B$.

Answer

Given, $a_n = An + B$

Put n = 1, $a_1 = A + B$

Put n = 20, $a_{20} = 20A + B$

$S_{20} = \frac{20}{2}[a + l]$

$= 10 [A + B + 20A + B]$

$= 10 [21A + 2B]$

$= 210A + 20B$

26. Question

Find the sum of first 25 terms of an A.P. whose nth term is given by $a_n = 2 - 3n$.

Answer

Given, nth term, a_n = 2 – 3n

Put n = 1, a_1 = 2 – 3(1) = -1

Put n = 25, a_{15}= 2 – 3(15) = -43

Therefore, $S_{25} = \frac{25}{2}$ (-1 – 43)

= $\frac{25}{2}$ (-44) = -925

27. Question

Find the sum of the first 25 terms of an A.P. whose nth term is given by $a_n = 7 - 3n$.

Answer

Given, a_n= 7 - 3n

Put n = 1

a_1= 7 – 3(1) = 4

Put n = 25

a_{25}= 7 – 3(25) = -68 = l

S_{25}= $\frac{25}{2}$ [4 – 68]

= 25 [-32] = -800

28. Question

Find the sum of the first 25 terms of an A.P. whose second and third terms are 14 and 18 respectively.

Answer

a_2= 14

a + d = 14 (i)

a_3= 18

a + 2d = 18 (ii)

Subtracting (i) from (ii), we get

d = 4

Putting the value of d in (i), we get

a = 14 – 4 = 10

$S_{25} = \frac{25}{2}[2(a) + 24(d)]$

= 25 [58]

= 1450

29. Question

If the sum of 7 terms of an A.P. is 49 and that of 17 term is 289, find the sum of n terms.

Answer

$S_7 = 49$

$\frac{7}{2}[2a + 6d] = 49$

a + 3d = 7 (i)

$S_{17} = 289$

$\frac{17}{2}[2a + 16d] = 289$

a + 8d = 17 (ii)

Subtract (i) from (ii), we get

5d = 10

d = 2

Put d = 2 in (i), we get

a = 7 – 6 = 1

$S_n = \frac{n}{2}[2(1) + (n - 1)2]$

= n [1 + n – 1]

$= n^2$

30. Question

The first term of an A.P. is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.

Answer

a = 5, l = 45

$S_n = 400$

$\frac{n}{2}[5 + 45] = 400$

$\frac{n}{2}[50] = 400$

n = 16

16^{th} term is 45

$a_{16} = 45$

5 + 15d = 45

15d = 40

$d = \frac{8}{3}$

31. Question

In an A.P., the sum of first n terms is $\frac{3n^2}{2} + \frac{13n}{2}$. Find its 25^{th} term.

Answer

Given, $S_n = \frac{3n^2}{2} + \frac{13}{2}n$

Let a_n be the n^{th} term of the A.P.

$a_n = S_n - S_{n-1}$

$= 3n^2/2 + 13n/2 - 3(n-1)^2/2 - 13(n-1)/2$

$= \frac{3}{2}\{n^2 - (n-1)^2\} + \frac{13}{2}\{n - (n-1)\}$

$= 3n - \frac{3}{2} + \frac{13}{2}$

$= 3n + \frac{10}{2} = 3n + 5$

Put n = 25

$a_{25} = 3(25) + 5$

= 75 + 5 = 80

Therefore, 25^{th} term is $a_{25} = 80$

32. Question

Let there be an A.P. with first term 'a', common difference, d. If a_n denotes its n^{th} term and S_n the sum of first n terms, find.

(i) n and S_n, if $a = 5, d = 3$ and $a_n = 50$.

(ii) n and a, if $a_n = 4, d = 2$ and $S_n = -14$.

(iii) d, if $a = 3, n = 8$ and $S_n = 192$.

(iv) a, if $a_n = 28, S_n = 144$ and $n = 9$.

(v) n and d, if $a = 8, a_n = 62$ and $S_n = 210$.

(vi) n and a_n, if $a = 2, d = 8$ and $S_n = 90$.

Answer

(i) n and S_n, if $a = 5, d = 3$ and $a_n = 50$.

Given, $a_n = 50$

$a + (n - 1) d = 50$

$5 + (n - 1) 3 = 50$

$(n - 1)3 = 45$

$(n - 1) = 15$

$n = 16$

$S_n = \frac{n}{2} [a + l]$

$= 8 [5 + 50] = 8 [55] = 440$

(ii) n and a, if $a_n = 4, d = 2$ and $S_n = -14$.

$a_n = 4$, $d = 2$, $S_n = -14$

$a + (n - 1) 2 = 4$

$a + 2n = 6$, and

$\frac{n}{2} [2a + (n - 1)2] = -14$

$n [2a + 2n - 2] = -14$, or

$\frac{n}{2} [a + a_n] = -14$

$\frac{n}{2} [a + 4] = -14$

$n [6 - 2n + 4] = -28$

$n [10 - 2n] = -28$

$2n^2 - 10n - 28 = 0$

$2(n^2 - 5n - 14) = 0$

$(n + 2) (n - 7) = 0$

$n = -2, n = 7$

Therefore, $n = -2$ is not a natural number. So, $n = 7$

(iii) d, if $a = 3, n = 8$ and $S_n = 192$.

Given, $a = 3$, $n = 8$, $S_n = 192$

$S_n = \frac{n}{2}$ {2a + (n – 1) d]

192 * 2 = 8 [6 + (8 – 1)d]

$\frac{192*2}{8}$ = 6 + 7d

48 = 6 + 7d

7d = 42

d = 6

(iv) a, if $a_n = 28, S_n = 144$ and $n = 9$.

a_n= 28, S_n = 144, n = 9

$S_n = \frac{n}{2}$ [a + l]

144 = $\frac{9}{2}$ [a + 28]

$\frac{144*2}{9}$ = a = 28

a + 28 = 32

a = 4

(v) n and d, if $a = 8, a_n = 62$ and $S_n = 210$.

Given, a = 8, a_n= 62 and S_n= 210

$S_n = \frac{n}{2}$ [a + l]

210 = $\frac{n}{2}$ [8 + 62]

210 * 2 = n (70)

n = $\frac{210*2}{40}$ = 6

a + (n – 1) d = 62

8 + (6 – 1) d = 62

5d = 54

d = 10.8

(vi) n and a_n, if $a = 2, d = 8$ and $S_n = 90$.

Given, a = 2, d = 8 and S_n = 90

90 = $\frac{n}{2}$ [4 + (n – 1)d] {Therefore, $S_n = \frac{n}{2}$ [2a + (n – 1)d]}

180 = n [4 + 8n – 8]

$8n^2 - 4n - 180 = 0$

$4(2n^2 - n - 45) = 0$

$2n^2 - n - 45 = 0$

$(2n + 1)(n - 5) = 0$

Therefore, $n = \frac{-1}{2}$ is not a natural number

$n = 5$

$a_n = a + (n - 1)d$

$= 2 + 4(8) = 32$

33. Question

Aman saved Rs. 16500 in ten years. In each year after the first he saved Rs.100 more than he did in the preceding year. How much did he save in the first year?

Answer

Given,

Man saved in 10 years, $S_{10} = 16500$

In each year after first he saved, d = 100

We know, sum of n terms, $S_n = \frac{n}{2}[2a + (n - 1)d]$

For 10 years

$S_{10} = \frac{10}{2}[2a + (10 - 1)100]$

$16500 = 5[2a + 9 * 100]$

$\frac{16500}{5} = 2a + 900$

$3300 = 2a + 900$

$2a = 3300 - 900$

$2a = 2400$

$a = 1200$

Hence, Rs. 1200 saved by him in first year

34. Question

Aman saved Rs.32 during the first year, Rs.36 in the second year and in this way he increases his savings by Rs.4 every year. Find in what time his saving will be Rs.200.

Answer

Given, A man saved in first year, a = 32

A man saved in second year, a_2 = 36

Increase saving, d = 4

In n years his saving will be 200, S_n = 200

We know,

$S_n = \frac{n}{2}$ [2a + (n − 1) d]

200 = $\frac{n}{2}$ [2(32) + (n − 1) 4]

400 = n [64 + 4n − 4]

400 = n [60 + 4n]

400 = 4n [15 + n]

100 = 15n + n^2

n^2 + 15n − 100 = 0

n^2 + 20n − 5n − 100 = 0

n (n + 20) − 5 (n + 20) = 0

(n − 5) (n + 20) = 0

Here, n − 5 = 0, n = 5

n + 20 = 0, n = -20

The term can never be negative. So, we consider n = 5

Hence, in 5 years his saving will be Rs. 200

35. Question

Aman arrange stop ay off a debt of Rs.3600 by 40 annual installments which for man arithmetic series. When 30 of the installments are paid, he dies leaving one - third of the debt unpaid, find the value of the first installment.

Answer

Given, Total amount of payable in 40 annual installments, S_{40} = 3600

After 30 installment he died and leaving $\frac{1}{3}$ of the debt unpaid

Which means, total in 30 installment, $S_{30} = \frac{2}{3}$ of the debt

$S_{30} = \frac{2}{3} * 3600 = 2400$

We know sum of n terms, $S_n = \frac{n}{2}$ [2a + (n − 1) d]

For 30 installments, $S_{30} = \frac{30}{2}[2a + (30 - 1)d]$

$2400 = 15[2a + 29d]$

$160 = 2a + 29d$

$2a = 160 - 29d$

$a = \frac{160-29d}{2}$ (i)

For 40 installments, $S_{40} = \frac{n}{2}[2a + (n - 1)d]$

$3600 = \frac{40}{2}[2a + (40 - 1)d]$

$180 = 2a + 39d$

$2a = 180 - 39d$

$a = \frac{180-39d}{2}$ (ii)

From (i) and (ii), we get

$\frac{160-29d}{2} = \frac{180-39d}{2}$

$160 - 29d = 180 - 39d$

$39d - 29d = 180 - 160$

$10d = 20$

$d = 2$

Putting the value of d in (i), we get

$a = \frac{160-29(2)}{2}$

$= \frac{160-58}{2} = \frac{102}{2} = 51$

Hence, value of first installment is 51

36. Question

There are 25 trees at equal distances of 5 metres in a line with a well, the distance of the well from the nearest tree being 10 metres. A gardener waters all the trees separately starting from the well and here turns to the well after watering each tree to get water for the next. Find the total distance the gardener will cover in order to water all the trees.

Answer

Total numbers of trees = 25

Distance between trees = 5

Total distance between well and first tree is = 10 m

Now, gardener return back to well after watering each tree then,

Distance between well and second tree is = 10 + 10 + 5 = 25

Distance between well and third tree = 25 + 5 + 5 = 35

Distance between well and fourth tree = 35 + 5 + 5 = 45

Distance between well and last tree without return back = 10 + 24(5) = 10 + 120 = 130

So, common difference = 35 – 25 = 10

Now, Total distance covered by 24 trees,

We knew, $S_n = \frac{n}{2}$ [2a + (n – 1) d]

$= \frac{24}{2}$ [2(25) + (24 – 1) 10]

= 12 [50 + 23 * 10]

= 12 [50 + 230]

= 12 * 280

= 3360

Now, distance covered by 25 trees = 10 + 3360 = 3370

Total distance covered by gardener with return back = 3370 + 130 = 3500 m

Hence, total distance will covered by gardener is 3500.

37. Question

A man is employed to count Rs.10710. He counts at there of Rs.180 per minute for half an hour. After this, he counts at the rate of Rs.3 less every minute than the preceding minute. Find the time taken by him to count the entire amount.

Answer

Total amount for counting = Rs. 10710

In 1 min he counts =Rs. 180

In half an hour (30 min) he count = Rs.180×30 =Rs.5400

Amount left count after half an hour, S_n= Rs.10710 –Rs. 5400 = 5310

Now, in 31st min he count 3 less than preceding minute = Rs.180 – Rs.3 = Rs.177

In 32nd min he count 3 less than preceding minute = Rs. 177 –Rs. 3 =Rs. 174

Then, Arithmetic progression formed is 177, 174,...

Here, difference between minutes = 174 – 177 = -3

We know, sum of n terms, S_n= (n/2) [2a + (n – 1) d]

$5310 = (n/2)\ [2(177) + (n - 1) -3]$

$5310 \times 2 = n\ [354 - 3n + 3]$

$10620 = 354n - 3n^2 + 3n$

$10620 = 357n - 3n^2$

$\Rightarrow 3n^2 - 357n + 10620 = 0$

On taking 3 common from the complete equation, we get,

$\Rightarrow 3\ (n^2 - 119n + 3540) = 0$

$\Rightarrow n^2 - 119n + 3540 = 0$

Now, from factoring by splitting the middle term method, we get,

$\Rightarrow n^2 - 60n - 59n + 3540 = 0$

$\Rightarrow n\ (n - 60) - 59\ (n -60) = 0$

$\Rightarrow (n - 59)\ (n - 60) = 0$

Therefore, either n = 59 or n = 60We will use 59 as these are minutesSo The total time to calculate whole amount is 59 + 30 = 89 min

= 1 hr 29 min

38. Question

A piece of equipment cost a certain factory Rs.600,000. If it depreciates in value, 15% the first, 13.5% the next year, 12% the third year, and soon. What will be its value at the end of 10 years, all percentages applying to the original cost?

Answer

Given: A piece of equipment cost a certain factory Rs.600,000. If it depreciates in value, 15% the first, 13.5% the next year, 12% the third year, and soon.

To find: The value at the end of 10 years if all percentages applying to the original cost.

Solution: Cost of equipment = 6, 00, 000

In 1 year the value depreciate by 15%

⇒ The value of the equipment after first year= $6,00,000 * \frac{15}{100} = 90,000$

In 2 year depreciate by 13.5%

⇒ The value of the equipment after second year= $6,00,000 * \frac{13.5}{100} = 81,000$

In 3 year depreciate by 12%

⇒ The value of the equipment after third year= $6,00,000 * \frac{12}{100} = 72,000$

Now A.P. is 90000, 81000, 72000,...

So, common difference = 81000 – 90000 = -9, 000

We have to find total depreciation for 10 years,Since the value of depreciation is constant in any consecutive years i.e its value is -9000. So to find the depreciation after ten years we will use the formula:$S_n = \frac{n}{2}$ [2a + (n – 1) d]

$S_{10} = \frac{10}{2}$ [2(90, 000) + (10 – 1) (-9000)]

S_{10}= 5 [180000 + 9 * (-9000)]

S_{10}= 5 [180000 – 81000]

S_{10}= 5 [99000] = 495000

Hence, cost of equipment at the end of 10 years = original cost – depreciation

= 6, 00, 000 – 4, 95, 000 = Rs. 1, 05, 000

39. Question

A sum of Rs.700 is to be used to gives even cash prizes to students of a school for their overall academic performance. If each prize is Rs.20 less than its preceding prize, find the value of each prize.

Answer

Amount of money = 700

Total number of prize = 7

Each prize is Rs. 20 less than its preceding prize

Let, Prize of first prize = a

Prize of second prize = a – 20

Prize of third prize = a – 20 – 20 = a – 40

Here, A.P. is a, a – 20, a – 40,…

So, common difference, d = a – 20 – a = -20

We know, $S_n = \frac{n}{2}$ [2a + (n – 1) d]

700 = $\frac{7}{2}$ [2a + (7 – 1) -20]

$\frac{1400}{7}$ = 2a + 6 (-20)

$2a = 200 + 120$

2a = 320

a = 160

So, value of first prize = 160

Value of second prize, 160 – 20 = 140

Value of third prize, 140 – 20 = 120

Value of fourth prize, 120 – 20 = 100

Value of fifth prize, 100 – 20 =80

Value of sixth prize, 80 – 20 = 60

And value of seventh prize, 60 – 20 = 40

Hence, value of prizes, Rs 160, Rs 140, Rs 120, Rs 100, Rs 80, Rs 60, Rs 40.

40. Question

In an A.P. the first term is 8, nth term is 33 and the sum to first n terms is 123. Find n and d, the common differences.

Answer

Given, First term, a = 8

N^{th} term, $a_n = 33 = l$

And sum of n terms, $S_n = \frac{n}{2}[a + l]$

$123 = \frac{n}{2}[8 + 33]$

123 * 2 = n * 41

$n = \frac{123*2}{41} = \frac{246}{41} = 6$

We know,

$a_n = a + (n - 1)\,d$

33 = 8 + (6 – 1) d

33 – 8 = 5d

25 = 5d

d = 5

Hence, number of terms n = 6 and common difference, d = 5

41. Question

In an A.P., the first term is 22, nth term is -11 and the sum to first n terms is 66. Find n and d, the common difference.

Answer

Given, First term, a = 22

n^{th} term, $a_n = -11 = l$

and sum of n terms, $S_n = 66$

We know sum of n terms, $S_n = \frac{n}{2}[a + l]$

$66 = \frac{n}{2}[22 - 11]$

$66 * 2 = n * 11$

$n = \frac{66*2}{11} = 12$

we know, $a_n = a + (n - 1) d$

$-11 = 22 + (12 - 1) d$

$-11 - 22 = 11d$

$-33 = 11d$

$d = -3$

Hence, number of terms, n = 12 and common difference, d = -3

42. Question

If the sum of the first n terms of an A.P. is $4n - n^2$, which is the first term? What is the sum of first two terms? What is the second term? Similarly, find the third, the tenth and the nth terms.

Answer

Given, $S_n = 4n - n^2$

Putting n = 1, 2

$S_1 = 4(1) - (1)^2 = 3$

$S_2 = 4(2) - (2)^2 = 4$

We know, $a_n = S_n - S_{n-1}$

For first term, $a_1 = S_1 - S_0 = 3 - 0 = 3$

For term $a_2 = S_2 - S_1 = 4 - 3 = 1$

Now fore term a_3,

$S_2 = 4(2) - (2)^2 = 8 - 4 = 4$

$S_3 = 4(3) - (3)^2 = 12 - 9 = 3$

$a_3 = S_3 - S_2 = 3 - 4 = -1$

43. Question

The first and the last term of an A.P. are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?

Answer

Given,

First term, $a_1 = 17$

Last term, $a_n = 350 = l$

And difference, $d = 9$

We know,

$a_n = a + (n - 1)d$

$350 = 17 + (n - 1) (9)$

$350 = 8 + 9n$

$342 = 9n$

$n = 38$

We know sum of n terms, $S_n = \frac{n}{2} [a + l]$

$S_{38} = \frac{38}{2} [17 + 350]$

$= 19 * 367 = 6973$

Hence, number of terms, $n = 38$

Sum of n terms, $S_n = 6973$

44. Question

In an A.P., the first term is 2, the last term is 29 and the sum of the terms is 155. Find the common difference of the A.P.

Answer

Given, First term, $a = 2$

Last term, $a_n = l = 29$

And, Sum of terms, $S_n = \frac{n}{2} [a + l]$

$155 = \frac{n}{2} [2 + 29]$

$155 * 2 = n * 31$

$n = \frac{155*2}{31} = \frac{310}{31} = 10$

We know, $a_n = a + (n - 1) d$

$29 = 2 = (10 -1) d$

$29 - 2 = 9d$

$27 = 9d$

$d = 3$

Hence, common difference, $d = 3$

45. Question

In an A.P., the sum of first ten terms is – 150 and the sum of its next ten terms is-550. Find the A.P.

Answer

Given: Sum of first ten terms that is, $S_{10} = -150$ and Sum of next ten terms is -550.

To find: The A.P.

Solution: Let, first term = a

Last term = a_nWe know in an A.P,$a_n = a + (n - 1) d$

For n=10,

$a_n = a + (10 - 1) d$

$a_n = a + 9d$

We know, Sum of n terms

$S_n = \frac{n}{2} [a + l]$ ...... (1)

l and a_n means the last term.

Put the value of l in (1),So,

$S_{10} = \frac{10}{2} [a + a + 9d]$

Substitute the known values,

$-150 = 5 (2a + 9d)$

$-150 = 5 (2a + 9d)$$-150= 10a + 45d$

$10a = -150 - 45d$

$a = \frac{-150-45d}{10}$ ..... (2)

For next 10 terms, First term = a_{11}

Last term = a_{20}

Since $a_n = a + (n - 1) d$,

For 11^{th} term,$a_{11} = a + 10d$

For 20^{th} term,$a_{20} = a + 19d$

And $S'_n = \frac{n}{2}[a + l]$ ...... (3)

Let the sum of next 10 terms to be S'_{10} ,

Now a will be equal to a_{11} in this case and l will be equal to a + 19d

Put the value of a and l in the equation (3).$S'_{10} = \frac{10}{2}[a + 10d + a + 19d]$

Substitute the known values,

-550 = 5 [2a + 29d]

-550 = 10a + 145d

10a = -550 – 145d

$a = \frac{-550-145d}{10}$ .....(4)

From (2) and (4), we get

$\frac{-150-45d}{10} = \frac{-550-145d}{10}$

-150 – 45d = -550 – 145d

-45d + 145d = -550 + 150

100d = -400

d = -4

Put the value of d in (2), we get

$a = \frac{-150-45(-4)}{10}$

$a = \frac{-150+180}{10} = \frac{30}{10} = 3$

We know A.P is of the form,a_1 , a_2 ,.......... ,a_n

Where a_1 is the first term of an A.P,

d is the common difference.anda_n = a+ (n-1)dSo,

$a_1 = 3$

$a_2 = a + d = 3 + (-4) = -1$

$a_3 = a + 2d = 3 + 2(-4) = -5$

Conclusion : The A.P. is 3, -2, -5,...

and a = 3, d = -4

46. Question

Sum of the first 14 terms of an A.P. is 1505 and its first term is 10. Find its 25th term.

Answer

Given, a = 10

$S_{14} = 1505$

$S_{14} = \frac{n}{2}[2a + (n - 1) d]$

$1505 = \frac{14}{2}[2(10) + (14 - 1) d]$

1505 = 7 [20 + 13d]

215 = 20 + 13d

13d = 215 – 20

13d = 195

d = 15

Now, $a_{25} = a + (n - 1) d$

= 10 + (25 – 1) 15

= 10 + 24 * 15

= 10 + 360 = 370

Hence, value of 25^{th} term is 370

47. Question

The sum of first n terms of an A.P. is $5n^2 + 3n$. If its *m*th term is 168, find the value of m. Also, find the 20^{th} term of this A.P.

Answer

Given that

$S_n = 5n^2 + 3n$

Put n = 1

$S_1 = T_1 = 5 + 3 = 8$

Put n = 2

$S_2 = 5(2)^2 + 3 + 2 = 26$

$T_2 = S_2 - S_1 = 26 - 8 = 18$

$S_3 = 5(3)^2 + 3 + 3 = 54$

$T_3 = S_3 - S_2 = 54 - 26$

= 28

Therefore, first term, a = 8 and common difference = 18 – 8 = 10

$T_m = a + (m - 1) d$

168 = 8 + (m – 1) 10

168 = 8 + 10m – 10

170 = 10m

m = 17

$T_{20} = 8 + (20 - 1) 10$

= 8 + 19 * 10 = 198

48. Question

The sum of first q terms of an A.P. is $63q - 3q^2$. If its pth term is-60, find the value of p. Also, find the 11^{th} term of this A.P.

Answer

Given that: $S_q = 63q - 3q^2$

Put q = 1

$S_1 = T_1 = 63 - 3 = 60$

Put q = 2

$S_2 = 63 * 2 - 3 * (2)^2$

= 126 – 12 = 114

$T_2 = S_2 - S_1 = 114 - 60 = 54$

Put q = 3

$S_3 = 63 * 3 - 3(3)^2$

= 189 – 27 = 162

$T_3 = S_3 - S_2$

= 162 – 114 = 48

Therefore, first term of this A.P. is 60 and Common difference is 54 - 60 = -6

$T_p = a + (p - 1) d$

-60 = 60 + (p – 1) (-6)

-120 = (p – 1) (-6)

(p – 1) = 20

p = 21

Now 11^{th} term of this A.P

$T_{11} = 60 + (11 - 1)(-6)$

$= 60 - 60 = 0$

49. Question

The sum of first m terms of an A.P. is 4 m^2 - m. If its nth term is 107, find the value of n. Also, find the 21^{st} term of this A.P.

Answer

$S_m = 4m^2 - m$

Put m = 1

$S_1 = T_1 = 4 - 1 = 1$

Put m = 2

$S_2 = 4(2)^2 - 2 = 14$

$T_2 = S_2 - S_1 = 14 - 3 = 11$

Put m = 3

$S_3 = 4(3)^2 - 3 = 33$

$T_3 = S_3 - S_2$

$= 33 - 14 = 19$

The first term of given A.P. is 3 and common difference, d = 11 − 3 = 8

n^{th} term of the given A.P. is 107

$107 = 3 + (n - 1)8$

$104 = (n - 1)8$

$(n - 1) = 13$

n = 14

the 21^{st} term of the given A.P., $T_{21} = 3 + (21 - 1)8$

$= 3 + 160 = 163$

50. Question

The nth term of an A.P. is given by (-4n+15). Find the sum of first 20 terms of this A.P.

Answer

Given: The nth term of an A.P. (-4n+15).

To find: the sum of first 20 terms of this A.P.

Solution: We have,

$T_n = (-4n + 15)$

$T_1 = -4 + 15 = 11$

$T_2 = (-4 \times 2) + 15 = 7$

$T_3 = (-4 \times 3) + 15 = 3$Hence the A.P is 11,7,3,.........

The first term is 11 and the common difference is, $d = T_2 - T_1 = 7 - 11 = -4$

We calculate the sum of terms of A.P by using the formula:$S_n = \frac{n}{2} [2a + (n - 1) d]$

Substitute the known values to get the sum.

The sum of first 20 terms, $S_{20} = \frac{20}{2} [(2 \times 11) + (20 - 1) (-4)]$

$S_{20} = \frac{20}{2} [(22) + (19) (-4)]$

$S_{20} = 10 (22 - 76)$ $S_{20} = 10(-54)$$S_{20} = -540$

51. Question

Find the number of terms of the A.P. If 1 is added to each term of this A.P., then find the sum of all terms of the A.P. thus obtained.

Answer

The given A.P. is -12, -9, -6,...21

Here, a = -12, d = -9 – (-12) = 3

Let the number of terms be n

$T_n = a + (n - 1) d$

21 = -12 + (n – 1) (3)

21 = -15 + 3n

36 = 3n

n = 12

If 1 is added to each term than A.P. is

-11, -8, -5...20

Here, a = -11, d = 3

$S_{12} = \frac{12}{2} [2(-11) + (12 -1) * 3]$

= 6 (-22 + 33)

= 66

52. Question

The sum of first n terms of an A.P. is $3n^2 + 4n$. Find the 25^{th} term of this A.P.

Answer

We have sum of n terms, $S_n = 3n^2 + 4n$

Put n = 1

$S_1 = T_1 = 3(1)^2 + 4(1) = 7$

Put n = 2

$S_2 = 3(2)^2 + 4(2) = 20$

$T_2 = S_2 - S_1 = 20 - 7 = 13$

Put n = 3

$S_3 = 3(3)^2 + 4(3) = 39$

$T_3 = S_3 - S_2 = 39 - 20 = 19$

Therefore, first term is 7 and common difference, d = 13 – 7 = 6

The 25^{th} term is, $T_n = a + (n - 1) d$

$T_{25} = 7 + (25 - 1) * 6$

= 7 + 24 * 6 = 151

53. Question

In a school, students decide to plant trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be double of the class in which they are studying. If there are 1 to 12 classes in the school and each class has two sections, find how many trees were planted by the students.

Answer

Each class has to plant double the class they are studying

Therefore, first class will plant = 2

Second class will plant = 4

Same as, 12^{th} class will plant = 24

Total number of plants to be planted = sum of all plants

$= \frac{n}{2}[a + l]$

$= \frac{12}{2}[2 + 24]$

= 6 * 26 = 156

Therefore, there are 2 sections in each class

So, total trees = 2 * 156 = 312

54. Question

The sum of first seven terms of an A.P. is 182. If its 4^{th} and the 17^{th} terms are in the ratio 1:5, find the A.P.

Answer

$S_7 = \frac{7}{2} [2a + (7 - 1) d]$

$182 = \frac{7}{2} [2a + 6d]$

52 = 2a + 6d

a + 3d = 26 (i)

Acc. To question,

$a_{17} = 5a_4$

a + 16d = 5(a + 3d)

d = 4a

d = 4(26 – 3d) [from (i)]

13d = 104

d = 8

Putting the value of d in (i), we get

a = 26 – 3*8) = 2

$a_1 = a = 2$

$a_2 = a + d = 2 + 8 = 10$

$a_3 = a + 2d = 2 + 2(8) = 18$

Hence, The A.P. is 2, 10, 18,...

55. Question

The sum of the first n terms of an A.P. is $3n^2 + 6n$. Find the nth term of this A.P.

Answer

Let ‘a’ be the first term and ‘d’ be the common difference

$S_n = 3n^2 + 6n$

Firsdt term, a, $S_1 = 3(1)^2 + 6(1)$

$= 3 + 6 = 9$

$S_2 = 3(2)^2 + 6(2)$

$a + a + d = 12 + 12$

$9 + 9 + d = 24$

$18 + d = 24$

$d = 6$

Therefore, $a_n = a + (n - 1) d$

$= 9 + (n - 1) 6$

$= 6n + 3$

56. Question

The sum of the first 7 terms of an A.P. is 63 and the sum of its next 7 terms is 161. Find the 28th term of this A.P.

Answer

$S_7 = 63$

$\frac{7}{2}[2(a) + 6d] = 639$

$a + 3d = 9$ (i)

$a8 = a + 7d$

Hence, for next 7 terms first term will be the 8th term i.e. $a + 7d$

Sum of next 7 terms, $S'_7 = \frac{7}{2}[2(a + 7d) + 6d]$

$161 = 7 [a + 7d + 3d]$

$23 = a + 10d$

$23 = 9 - 3d + 10d$ [From (i)]

$14 = 7d$

$d = 2$

Putting the value of d in (i), we get

$A = 9 - 3(2) = 3$

Now, $a_{28} = a + 27d$

$= 3 + 27(2)$

$= 3 + 54 = 57$

57. Question

If demotes the sum of the first n terms of an A.P., prove that S_{30}= 3 (S_{20} – S_{10}) .

Answer

Proof: $S_{20} = \frac{20}{2}$ [2a + 19d]

= 10 [2a + 19d] (i)

$S_{10} = \frac{10}{2}$ [2a + 9d]

= 5 [2a + 9d] (ii)

L.H.S: $S_{30} = \frac{30}{2}$ [2a + 29d]

= 15 [2a + 29d]

R.H.S: 3 (S_{20} – S_{10})

= 3 [10 (2a + 19d) – 5 (2a + 9d)] {From (i) and (ii)}

= 15 [4a + 38d – 2a – 9d]

= 15 [2a + 29d]

Since, L.H.S = R.H.S

Hence, proved

58. Question

The sum of first n terms of an A.P. is $5n - n^2$. Find the nth term of this A.P.

Answer

$S_n = 5n - n^2$

First term, a = $5(1) - (1)^2$

= 5 – 1 = 4

$S_2 = 5(2) - (2)^2$

a + (a + d) = 10 – 4

4 + 4 + d = 6

d = -2

Now, the n^{th} term, a_n = a + (n - 1) d

= 4 + (n – 1) (-2)

= 4 + 2 – 2n = 6 – 2n

59. Question

The sum of the first n terms of an A.P. is $4n^2 + 2n$. Find the nth term of this A.P.

Answer

$S_n = 4n^2 + 2n$

First term, $a = 4(1)^2 + 2(1)$

$= 4 + 2 = 6$

Sum of first two terms, $S_2 = 4(2)^2 + 2(2)$

$a + a + d = 16 + 4$

$6 + 6 + d = 20$

$d = 8$

Now the n^{th} term, $a_n = a + (n - 1) d$

$= 6 + (n - 1) 8$

$= 6 + 8n - 8$

$= 8n - 2$

60. Question

If the 10^{th} term of an A.P. is 21 and the sum of its first ten terms is 120, find its nth term.

Answer

$a_{10} = a + 9d$

$a + 9d = 21$ (i)

$S_{10} = \frac{10}{2} [2a + 9d]$

$120 = 5 [2a + 9d]$

$24 = 2a + 9d$

$24 = 2 (21 - 9d) + 9d$ [From (i)]

$-18 = -9d$

$d = 2$

Putting the value of d in (i), we get

$a = 21 - 9(2)$

$= 3$

The n^{th} term, $a_n = a + (n - 1) d$

$= 3 + (n - 1) 2$

$= 3n + 2n - 2$

= 2n + 1

61. Question

Ram kali would need Rs.1800 for admission fee and books etc., for her daughter to start going to school from next year. She saved Rs.50 in the first month of this year and increased her monthly saving by Rs.20. After a year, how much money will she save? Will she be able to fulfill her dream of sending her daughter to school?

[CBSE2005,2014]

Answer

a = 50, d = 20

a_n = a + (n − 1) d = l

a_{12} = 50 + 11 * 20 = 270

$S_n = \frac{n}{2}$ [a + l]

$= \frac{12}{2}$ [50 + 270]

= 12 * 160 = 1920

Yes, she will be able to fulfill her dream of sending her daughter to school.

62. Question

The first and the last terms of an A.P. are 5 and 45 respectively. If the sum of all its terms is 400, find its common difference.

Answer

First term, a = 5

Last term, l = 45

S_n = 400

$\frac{n}{2}$ [a + l] = 400

$n\ [5 + 45] = 800$

n = 16

Now, $n = \frac{l-a}{d} + 1$

$16 = \frac{45-5}{d} + 1$

16d = 40 + d

15d = 40

$d = \frac{8}{3}$

Hence, the common difference is $\frac{8}{3}$

63. Question

The first and the last terms of an AP are 7 and 49 respectively. If sum of all its terms is 420, find its common difference.

Answer

First term, a = 7

Last term, l = 49

$S_n = \frac{n}{2}[a + l]$

$420 = \frac{n}{2}[7 + 49]$

840 = 56n

n = 15

Now, $n = \frac{l-a}{d} + 1$

$15 = \frac{49-7}{d} + 1$

15d = 42 + d

14d = 42

d = 3

Hence, it's common difference is 3

64. Question

If demotes the sum of first n terms of an A.P., prove that $S_{12} = 3(s_8 - S_4)$.

Answer

$S_8 = \frac{8}{2}[2a + 7d]$

= 4 (2a + 7d) (i)

$S_4 = \frac{4}{2}[2a + 7d]$

= 2 (2a + 7d) (ii)

L.H.S = $S_{12} = \frac{12}{2}[2a + 11d]$

= 6 [2a + 11d]

R.H.S = $3(S_8 - S_4)$

= 3 [4 (2a + 7d) – 2 (2a + 3d)] [From (i) and (ii)]

= 6 [4a + 14d − 2a − 3d]

= 6 [2a + 11d]

Since, L.H.S = R.H.S

Hence, proved

65. Question

If the sum of first n terms of an A.P. is $\frac{1}{2}(3n^2 + 7n)$, then find its nth term. Hence write its 20th term.

Answer

$S_n = \frac{1}{2}(3n^2 + 7n)$

First term, $a = S_1 = \frac{1}{2}[3(1)^2 + 7(1)]$

$= \frac{1}{2}[3 + 7] = 5$

$S_2 = \frac{1}{2}[3(2)^2 + 7(2)]$

$a + a + d = \frac{1}{2}[12 + 14]$

5 + 5 + d = 13

d = 3

$a_n = a + (n - 1)d$

= 5 + (n − 1) 3

= 5 + 3n − 3

= 3n + 2

$a_{20} = 3(20) + 2$

= 60 + 2 = 62

Hence, it's 20th term is 62

66. Question

The sum of first 9 terms of an A.P. is 162. The ratio of its 6th term to its 13th term is 1:2. Find the first and 15th term of the A.P.

Answer

Let a be the first term and d be the common difference

Now, $a_6 / a_{13} = \frac{1}{2}$

$\frac{a+5d}{a+12d} = \frac{1}{2}$

$2a + 10d = a + 12d$

$a = 2d$ (i)

Now sum of first n terms of A.P

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$S_9 = \frac{9}{2}[2a + 8d]$

$162 = 9(a + 4d)$

$a + 4d = 18$

$2d + 4d = 18$ [Using (i)]

$6d = 18$

$d = 3$

Now from (i), we get

$a = 2 * 3 = 6$

So, first term, $a_1 = a = 6$

15^{th} term, $a_{15} = a + 14d = 6 + 14(3)$

$= 6 + 42 = 48$

Therefore, First term is 6 and 15^{th} term is 48

CCE - Formative Assessment

1. Question

Define an arithmetic progression.

Answer

An arithmetic progression is a list of numbers in which each term is obtained by adding a fixed number to the preceding term except the first term. This fixed number is called the common difference of the AP.

2. Question

Write the common difference of an A.P. whose nth term is $a_n = 3n + 7$.

Answer

If $a_n = 3n + 7$

Then $a_1 = 3(1) + 7 = 10$

$a_2 = 3(2) + 7 = 13$

$a_3 = 3(3) + 7 = 16$

d= common difference = $a_n - a_{n-1}$

$= a_3 - a_2$

= 16 – 13 = 3

3. Question

Which tem of the sequence 114, 109, 104, ... is the first negative term?

Answer

Here a = 114, d is common difference

$d = a_3 - a_2 = a_2 - a_1 = -5$

For finding first negative term

$T_n < 0$

$a + (n - 1) d < 0$

$114 + (n - 1) (- 5) < 0$

$114 - 5n + 5 < 0$

$119 - 5n < 0$

$- 5n < - 119$

$5n > 119$

$n > 119/5$

$n > 23.8$

Therefore first negative term is 24^{th} term

4. Question

Write the value of $a_{30} - a_{10}$ for the A.P. 4, 9, 14, 19, ...

Answer

Here $a_1 = 4$

$a_2 = 9$

$a_3 = 14$

d = common difference = $a_3 - a_2 = a_2 - a_1 = 14 - 9 = 9 - 4 = 5$

$a_{30} = a + (n-1) d$

$a_{30} = 4 + (30 - 1) 5$

$a_{30} = 4 + 29 X 5$

$a_{30} = 149$

Similarly, $a_{10} = 4 + 9x5$

$a_{10}= 49$

$a_{30} - a_{10} = 149 - 49 = 100$

5. Question

Write 5th term from the end of the A.P. 3, 5, 7, 9, ..., 201.

Answer

Here $a = 201, a_2 = 5, a_3 = 7$

$d = a_3 - a_2 = a_2 - a_1$

$= 7 - 5 = 5 - 3 = 2$

$t_n = a + (n - 1)d$

$t_n = 3 + (n-1)2 = 201$

$(n - 1) 2 = 201 - 3 = 198$

$n - 1 = \frac{198}{2} = 99$

$n = 99 + 1 = 100$

5^{th} term from end = 96th term

$T_{95} = 3 + (96 - 1)2$

$T_{95} = 3 + 95 \times 2$

$T_{95} = 3 + 190 = 193$

6. Question

Write the value of x for which 2x, x + 10 and 3x + 2 are in A.P.

Answer

Terms are in A.P. if common difference (d) is same between two continuous numbers.

$x + 10 - 2x = 3x + 2 - (x + 10)$

$10 - x = 2x - 8$

$18 = 3x$

$x= 6$

So the terms in A.P. are: 2(6), 6 + 10, 3(6) + 2

= 12, 16, 20

7. Question

Write the nth term of an A.P. the sum of whose n terms is S_n.

Answer

First term = a

Sum up to first term = a

Last term (nth term) = a_n

Sum up to n terms = S_n

Second last term = a_{n-1}

Sum up to $(n-1)^{th}$ term = s_{n-1}

Therefore, $a_n = S_n - S_{n-1}$

8. Question

Write the sum of first n odd natural numbers.

Answer

First n odd numbers are 1, 3, 5, 7, 9........., (2n−1) which forms an A.P

a= 1

l= 2n – 1 where l be the last term

$S_n = \frac{n}{2}(2a + (n-1)d)$ or $n/2(a + l)$

$= n/2\ (1 + 2n - 1)$

$= \frac{n}{2}(2n)$

$= n\ (n)$

$= n^2$

9. Question

Write the sum of first n even natural numbers.

Answer

First n even Natural numbers are 2, 4, 6,......, 2n

This forms an A.P where

a= 2

l= 2n where l is last term of the A.P

$S_n = n/2\ (2a + (n-1)d)$ or $n/2\ (a + l)$

$= n/2\ (2 + 2n)$

$=n/2\ [2(1 + n)]$

$=n\ (n + 1)$

10. Question

1f the sum of n terms of an A.P. is $S_n = 3n^2 + 5n$. Write its common difference.

Answer

Let a1, a2, a3........ an be the given A.P

Given, sum of n terms = $3n^2 + 5n$

$S_n=3n^2 + 5n$..........(1)

Putting n =1 in (1)

$S_n= 3x1^2 + 5x1$

= 3 + 5 = 8

Sum of first 1 terms = first term

∴ first term = a = S_1 = 8

$S_n = 3n^2 + 5n$

Putting n= 2 in..... (1)

$S_2= 3x\ 2^2 + 5x2$

S_2= 22

Sum of first two terms = first term + second term

$S_2= a_1 + a_2$

$S_2 - a_1 = a_2$

a_2= 22 – 8 = 14

Thus a_1= 8, a_2 = 14

d= common difference = 14 – 8= 6

11. Question

Write the expression for the common difference of an A.P. whose first term is a and nth term is b.

Answer

The n^{th} term of the A.P whose first term a_1 and common difference is d is given by

$a_n = a + (n-1)\ d$

Here a_n is given as b

so $b = a + (n-1) d$

$\frac{b-a}{n-1} = d$

$\Rightarrow d = \frac{b-a}{n-1}$

12. Question

The first term of an A.P. is p and its common difference is q. Find its 10th term.

Answer

Given: The first term of an A.P. is p and its common difference is q.

To find: its 10th term.

Solution: The first term of A.P is p.Common difference = q

We know $a_n = a + (n-1)d$Where a is first term and d is common difference.

So,$a_{10} = p + (10-1)q$

$a_{10} = p + 9q$

13. Question

For what values of p are 2p + 1, 13, 5p − 3 are three consecutive terms of an A.P.?

Answer

Given: 2p + 1, 13, 5p − 3 are three consecutive terms of an A.P.

To find: The value of p

Solution: Consider $a_1 = 2p + 1$, $a_2 = 13$ and $a_3 = 5p - 3$The A.P will be of the form 2p + 1, 13, 5p − 3,.........Since the terms are in A.P so the common differences in them is same.$\Rightarrow a_2 - a_1 = a_3 - a_2$

$\Rightarrow 13 - (2p + 1) = 5p - 3 - 13 \Rightarrow 13 - 2p - 1 = 5p - 3 - 13$

$\Rightarrow 12 - 2p = 5p - 16$

$\Rightarrow 12 + 16 = 5p + 2p$

$\Rightarrow 28 = 7p \Rightarrow p = 4$

14. Question

If $\frac{4}{5}$, a, 2 are three consecutive terms of an A.P., then find the value of a.

Answer

$\frac{4}{5}$, a, 2 are three terms in an A.P so there common difference will be same

$a - 4/5 = 2 - a$

$2a = 2 + \frac{4}{5}$

$2a = \frac{14}{5}$

$a = 7/5$

15. Question

If the sum of first p term of an A.P. is $ap^2 + bp$, find its common difference.

Answer

Here, $s_p = ap^2 + bp$

$S_1 = a + b$

$S_2 = ax2^2 + bx2$

$= 4a + 2b$

$S_2 = a_1 + a_2$

$4a + 2b = a_1 + a_2$

$A_2 = 4a + 2b - (a + b)$

$A_2 = 3a + b$

Now, d = common difference = $a_2 - a_1$

$= 3a + b - (a + b)$

$= 2a$

1. Question

If 7th and 13th terms of an A.P. be 34 and 64 respectively, then its 18th term is

A. 87

B. 88

C. 89

D. 90

Answer

Here, $a_7 = 34$

$a_{13} = 64$

$a_7 = a + 6d = 34$……….(1)

a_{13} = a + 12d = 64............(2)

Subtracting (1) from (2)

6d = 30

D = 5

Multiplying (1) by 2

2a + 12d = 68.....................(3)

Subtracting (2) from (3)

a = 4

a_{18} = a + (n−1) d

a_{18} = 4 + (17) 5

a_{18} = 89 = C

2. Question

If the sum of P terms of an A.P. is q and the sum of q terms is p, then the sum of p + q terms will be

A. 0

B. p − q

C. p + q

D. − (p + q)

Answer

Let a be the first term and d is common difference of the A.P

then sum of n terms in A.P is $S_n = (\frac{n}{2})[\ 2a + (n - 1)\ d]$

Here, $s_p = q$

$S_q = p$

$S_p = (\frac{p}{2})\ [2a + (p - 1)\ d]$

$q = \frac{p}{2}\ [2a + (p - 1)\ d]$

$\frac{2q}{p} = [2a + (p - 1)\ d]$ --------(1)

$S_q = p = (\frac{q}{2})\ [2a + (q - 1)\ d]$

$\frac{2p}{q} = [2a + (q - 1)\ d]$ ....(2)

Subtract (1) from (2) we get

$$(q - p)d = \left(\frac{2p}{q}\right) - \left(\frac{2q}{p}\right)$$

(q – p) d = $(2p^2 - 2q^2)$ / pq -----------(3)

d = -2(q + p) / pq -----------(3)

Sum of first (p + q) terms

$$Sp + q = \frac{p + q}{2}[2a + (p + q - 1) d]$$

$$Sp + q = \frac{p + q}{2}[2a + (p - 1) d + qd]$$

$$Sp + q = \frac{p+q}{2}[\left(\frac{2q}{p}\right) + q\left(-\frac{2(q+p)}{pq}\right)]$$

[from (1) and (3)]

$$Sp + q = \frac{p + q}{2}\left[\frac{2q - 2q - 2p}{p}\right)$$

$$Sp + q = \frac{p + q}{2}\frac{(-2p)}{p}$$

$$Sp + q = -(p + q)$$

3. Question

If the sum of n terms of an A.P. be $3n^2 + n$ and its common difference is 6, then its first term is

A. 2

B. 3

C. 1

D. 4

Answer

Here, $S_n = 3n^2 + n$

d = 6

Putting n= 1

$S_1 = 3 + 1 = 4$

Sum of first 1 term = first term = 4

4. Question

The first and last terms of an A.P. are 1 and 11. If the sum of its terms is 36, then the number of terms will be

A. 5

B. 6

C. 7D. 8

Answer

a_1 = 1 (first term)

a_{11} = 11 = l (last term)

S_n= 36

But, $S_n = \frac{n}{2}(a + l)$

$36 = \frac{n}{2}(12)$

n= 6

5. Question

If the sum of n terms of an A.P. is $3n^2 + 5n$ then which of its terms is 164?

A. 26^{th}

B. 27^{th}

C. 28^{th}

D. none of these

Answer

Here, $S_n = 3n^2 + 5n$

$S_1 = a_1 = 3 + 5 = 8$

$S_2 = a_1 + a_2 = 12 + 10 = 22$

$\Rightarrow a_2 = S_2 - S_1 = 22 - 8 = 14$

$S_3 = a_1 + a_2 + a_3 = 27 + 15 = 42$

$\Rightarrow a3 = S3 - S2 = 42 - 22 = 20$

∴ Given AP is 8, 14, 20, ...

Thus a = 8, d = 6

Given t_m = 164.

164 = [a + (n −1)d]

$164 = [(8) + (m - 1)6]$

$164 = [8 + 6m - 6]$

$164 = [2 + 6m]$

$162 = 6m$

$m = 162 / 6.$

$\therefore m = 27.$

6. Question

If the sum of n terms of an A.P. is $2n^2 + 5n$, then its nth term is

A. $4n - 3$

B. $3n - 4$

C. $4n + 3$

D. $3n + 4$

Answer

Here $S_n = 2n^2 + 5n$

Sum of the A.P with 1 term = $S_1 = 2 + 5 = 7$ = first term

Sum of the A.P with 2 terms = $8 + 10 = 18$

Sum of the A.P with 3 terms = $18 + 15 = 33$

$a_2 = S_2 - S_1 = 18 - 7 = 11$

$d = a_2 - a_1 = 11 - 7 = 4$

n^{th} term = $a + (n-1) d$

$= 7 + (n-1) 4$

n^{th} term = $4n + 3$

7. Question

If the sum of three consecutive terms of an increasing A.P. is 51 and the product of the first and third of these terms is 273, then the third term is

A. 13

B. 9

C. 21

D. 17

Answer

Let 3 consecutive terms A.P is a −d, a, a + d. and the sum is 51

So, (a −d) + a + (a + d) = 51

3a −d + d = 51

3a = 51

a = 17

The product of first and third terms = 273

So, (a −d) (a + d) = 273

$a^2 - d^2 = 273$

$17^2 - d^2 = 273$

$289 - d^2 = 273$

$d^2 = 289 - 273$

$d^2 = 16$

d = 4

Third term = a + d = 17 + 4 = 21

8. Question

If four numbers in A.P. are such that their sum is 50 and the greatest number is 4 times the least, then the numbers are

A. 5, 10, 15, 20

B. 4, 10, 16, 22

C. 3, 7, 11, 15

D. none of these

Answer

Let the 4 numbers be a, a + d, a + 2d, a + 3d.

Sum of 4 numbers in A.P = 50

a + a + d + a + 2d + a + 3d = 50

⇒ 4a + 6d = 50

⇒ 2a + 3d = 25 ---------------(1)

Given the greatest number is 4 times the least.

4(a) = a + 3d

4a − a = 3d

a = d

Putting, a = d in (1), we obtain

5d = 25

d = 5

⇒a = 5

∴ First four terms are 5, 10, 15, 20.

9. Question

Let S_n denote the sum of n terms of an A.P. whose first term is a. If the common difference d is given by $d = S_n - kS_{n-1} + S_{n-2}$, then k =

A. 1

B. 2

C. 3

D. none of these.

Answer

Let a be the first term, n be the number of terms and d be the common difference of AP.

Given $d = S_n - kS_{n-1} + S_{n-2}$.

Now let n = 3

So, AP is : a, a + d, a + 2d

And $d = S_3 - k\ S_{3-1} + S_{3-2}$

$d = S_3 - k\ S_2 + S_1$ ....(1)

Sum of n terms of an AP is given as:

$S_n = (\frac{n}{2}) \times \{2a + (n-1)\ d\}$

Now $S_1 = a$

$S_2 = (\frac{2}{2}) \times (2a + (2-1)\ d)$ (n =2)

$S_2 = (2a + d)$

$S_3 = (\frac{3}{2}) \times (2a + (3-1)d)$ (n =3)

$S_3 = \frac{3}{2} \times (2a + 2d)$

$S_3 = 3(a + d)$

$S_3 = 3a + 3d$

Putting values of S1, S2 and S3 in equation 1, we get

$d = 3a + 3d - k(2a + d) + a$

$d = 4a + 3d - k(2a + d)$

$k(2a + d) = 4a + 3d - d$

$k(2a + d) = 4a + 2d$

$k(2a + d) = 2(2a + d)$

$k = 2$

10. Question

The first and last term of an A.P. are a and l respectively. If S is the sum of all the terms of the A.P. and the common difference is given by $\frac{l^2 - a^2}{k - (l + a)}$, then k=

A. S

B. 2S

C. 3S

D. none of these

Answer

Let the common difference and number of terms of AP be d and n respectively.

Last term of AP = a_n = l (given)

$l = a + (n-1)d$

$n = \frac{l-a}{d} + 1$................. (1)

$S = \frac{n}{2}(2a + (n-1)d)$

$S = \frac{1}{2}(\frac{l-a}{d} + 1)(2a + (\frac{l-a}{d} + 1 - 1)d)$ .... using (1)

$S = \frac{1}{2}(\frac{l-a}{d} + 1)(2a + l - a)$

$S = \frac{1}{2}(\frac{l-a}{d} + 1)(a + l)$

$(\frac{l-a}{d} + 1) = \frac{2S}{a+l}$

$\frac{l-a}{d} = \frac{2S}{a+l} - 1$

$\frac{l-a}{d} = \frac{2S-(L+a)}{a+l}$

$D = \frac{(a+l)(a-l)}{2S-(L+a)}$

Comparing this with $\frac{l^2 - a^2}{k-(l+a)}$

We get k =2S

11. Question

If the sum of first n even natural numbers is equal to k times the sum of first n odd natural numbers, then k =

A. $\frac{1}{n}$

B. $\frac{n-1}{n}$

C. $\frac{n+1}{2n}$

D. $\frac{n+1}{n}$

Answer

Given: the sum of first n even natural numbers is equal to k times the sum of first n odd natural numbers.

To find: The value of k

Solution: Sum of terms of A.P = n/2 (2a + (n-1) d)First n even natural numbers are: 2, 4, 6, 8, ...

It forms an AP where first term a = 2 and common difference d = 4 – 2 = 2

⇒ sum of n even natural terms = $\frac{n}{2}$ x {2 x 2 + (n−1)2}= $\frac{n}{2}$ (4 + 2n - 2)= $\frac{n}{2}$ (2 + 2n)=$\frac{n}{2}$ × 2 (1 + n)

= n (n + 1)

First n odd natural numbers are: 1, 3, 5, 7, ...

It forms an AP where first term is a = 1 and the common difference d = 3 – 1 = 2

Now sum of n terms = $\frac{n}{2}${2 x 1 + (n−1)2}

= $\frac{n}{2}$ (2 + 2n - 2)= $\frac{n}{2}$ (2n)

= n x n

$= n^2$

Now, According to given condition

Sum of first n even numbers = k X (Sum of first n odd numbers)

$\Rightarrow n(n+1) = k \times n^2$

$\Rightarrow k \times n = n + 1$

$\Rightarrow k = \frac{(n+1)}{n}$

12. Question

If the first, second and last term of an A.P. are a, b and 2a respectively, its sum is

A. $\frac{ab}{2(b-a)}$

B. $\frac{ab}{b-a}$

C. $\frac{3ab}{2(b-a)}$

D. none of these

Answer

a, b and 2a are in A.P so a is the first term and 2a is the last term denoted by T and T_n respectively. Here Common difference = b − a

$T_n = 2a = a + (n-1)(b-a)$

So $n = \frac{b}{b-a}$

Sum $= \frac{n}{2}\{\text{first term} + \text{last term}\}$

$= \frac{b}{2(b-a)}\{3a\}$

$= \frac{3ab}{2(b-a)}$

13. Question

If S_1 is the sum of an arithmetic progression of 'n' odd number of terms and S_2 the sum of the terms of the series in odd places, then $\frac{S_1}{S_2} =$

A. $\frac{2n}{n+1}$

B. $\frac{n}{n+1}$

C. $\frac{n+1}{2n}$

D. $\frac{n+1}{n}$

Answer

$S_1 = \frac{n}{2}(2a + (n-1)\, d)$

Out of these odd numbers of terms, there are $\frac{n+1}{2}$ terms in odd places

$S_2 = \frac{n+1}{2 \times 2}(2a + (\frac{n+1}{2} - 1)\, d)$

Common difference of two odd places is 2d

$S_2 = \frac{n+1}{4}(2a + (n-1)\, d)$

Now,

$$\frac{S_1}{S_2} = \frac{\frac{n}{2}(2a + (n-1)\, d)}{\frac{n+1}{4}(2a + (n-1)\, d)}$$

$$\frac{S_1}{S_2} = \frac{n+1}{n}$$

14. Question

If in an A.P., $S_n = n^2p$ and $S_m = m^2p$, where S_r denotes the sum of r terms of the A.P., then S_p is equal to

A. $\frac{1}{2}p^3$

B. mn p

C. p^3

D. $(m + n)\, p^2$

Answer

Let first term = a and Common difference = d

∴ According to the question, $S_n = n^2 p$

$S_n = n/2\ (2a + (n-1)\ d) = n^2p$

$2a + (n-1)\ d = 2np$..................(1)

And $S_m = m^2p$

$S_m = m/2\ (2a + (m-1)\ d) = m^2p$

$2mp = (2a + (m-1)\ d)$..................(2)

Subtracting 2 from 1

$2a + (n-1)\ d - 2a - (m-1)\ d = 2\ np - 2\ mp$

$d\ (n-1 - m + 1) = 2p\ (n - m)$

$d = 2p$

putting value of d in (1)

$2a + (n-1)\ 2p = 2np$

$a + (n-1)p = np$

$a = p$

now $S_p = p/2\ (2a + (p-1)\ d)$

putting value of $a = p$ and $d = 2p$

$S_p = p/2\ (2p + (p-1)\ 2p)$

$S_p = p^3$

15. Question

If S_n denote the sum of the first n terms of an A.P. If $S_{2n} = 3S_n$, then $S_{3n} : S_n$ is equal to

A. 4

B. 6

C. 8

D. 10

Answer

We know that $S_n = \frac{n\,(n+1)}{2}$

Now it is given that

$S_{2n} = 3S_n$

$\frac{2n(2n+1)}{2} = 3(\frac{n(n+1)}{2})$

$2(2n + 1) = 3(n + 1)$

$4n + 2 = 3n + 3$

$n = 1$

now, $S_{3n}/S_n = (\frac{3n(3n+1)/2]}{n(n+1)/2})$

$= \frac{3 \times 4}{2}$

$= 6$

16. Question

In an AP, $S_p = q$, $S_q = p$ and S_r denotes the sum of first r terms. Then, S_{p+q} is equal to

A. 0

B. $-(p + q)$

C. $p + q$

D. pq

Answer

$S_p = \frac{p}{2}\{2A + (p-1) D\} = q$, given

$S_q = \frac{q\{}{2} (2A + (q-1) D\} = p$, given

On Subtracting the second equation from 1st we get,

$D = -2\frac{(p+q)}{pq}$

Also On adding the two equations we get,

$2A + \frac{(p+q-2)D}{2} = p/q + q/p$

Now,

$S_{p+q} = \frac{(p+q)}{2}\{2A + (p + q-1) D\}$

$=\frac{(p+q)}{2}\{2A + (p + q-1)D\}$

$\frac{(p+q)}{2}\{\frac{p}{q} + \frac{q}{p} - \frac{(p+q-2)D}{2} + (p + q-1) D\}$

$=\frac{(p+q)}{2}\{p2 + q2 - (p + q)2\}/pq$

[By substituting the value of D]

= – (p + q)

17. Question

If S_r denotes the sum of the first r terms of an A.P. Then, S_{3n}: (S_{2n} — S_n) is

A. n

B. 3n

C. 3

D. none of these

Answer

$S_{2n} = \frac{2n}{2}(2a + (2n-1)\ d)$

$S_n = \frac{n}{2}(2a + (n-1)d)$

$S_{3n} = \frac{3n}{2}(2a + (n-1)d)$

$S_{2n} - S_n = \frac{2n}{2}(2a + (2n-1)\ d) - \frac{n}{2}(2a + (n-1)d)$

$= \frac{1n}{2}(2a + (n-1)\ d)$

$S_{3n}: (S_{2n} - S_n) = \frac{3n}{2}(2a + (n-1)d) : \frac{1n}{2}(2a + (n-1)\ d)$

= 3

18. Question

If the first term of an A.P. is 2 and common difference is 4, then the sum of its 40 terms is

A. 3200

B. 1600

C. 200

D. 2800

Answer

Here $a_1 = 2$

D = common difference = 4

$S_n = \frac{n}{2}(2a + (n-1)\ d)$

$S_{40} = \frac{40}{2}(2 \times 2 + (39)\ 4)$

$= 20\ (4 + 156)$

$= 3{,}200$

19. Question

The number of terms of the A.P. 3, 7, 11, 15, ... to be taken so that the sum is 406 is

A. 5

B. 10

C. 12

D. 14

Answer

Here a = 3

$a_2 = 7$

$a_3 = 11$

d= a3− a2 = a2− a1 = 4

$S_n = {}^{n}/_{2}\ (2a + (n-1)\ d)$

$406 = {}^{n}/_{2}\ (6 + (n-1)4)$

$406 = {}^{n}/_{2}\ (4n + 2)$

$2n^2 + n - 406 = 0$

$2n^2 + 29\ n - 28\ n - 406 = 0$

$n\ (2n + 29) - 14(2n + 29) = 0$

$(n - 14)\ (2n + 29) = 0$

Number of terms cannot be negative and in fractions so n= 14

20. Question

Sum of n terms of the series $\sqrt{2} + \sqrt{8} + \sqrt{18} + \sqrt{32} + ..$ is

A. $\dfrac{n(n+1)}{2}$

B. 2n (n + 1

C. $\dfrac{n(n+1)}{\sqrt{2}}$

D. 1

Answer

The given A.P is $\sqrt{2}+\sqrt{8}+\sqrt{18}+\sqrt{32}+..$

The simplified A.P is

$\sqrt{2}, 2\sqrt{2}, 3\sqrt{2}, 4\sqrt{2}, \ldots..$

Here a= $\sqrt{2}$

d= $2\sqrt{2} - \sqrt{2} = \sqrt{2}$

$S_n = n/2\ (2a + (n-1)\ d)$

$= n/2\ (2\sqrt{2} + (n-1)\sqrt{2})$

$= \sqrt{2}n/2\ (1 + n)$

$= n\ (n + 1)/\sqrt{2}$

21. Question

The 9th term of an A.P. is 449 and 449th term is 9. The term which is equal to zero is

A. 501th

B. 502th

C. 508th

D. none of these

Answer

Here, $a_9 = 449$

$A_{449} = 9$

Let a is the first term and d is the common difference of the AP

Given 9th term of AP = 499

a + 8d = 499 .....(1)

Again 499th term of AP = 9

a + 498d = 9 .....(2)

Now subtract equation 1 and 2, we get

a + 8d – (a + 498d) = 499 – 9

a + 8d – a – 498d = 499 – 9

–490d = 490

d = −490/490

d = −1

Put value of d in equation1, we get

a − 8 = 499

a = 499 + 8

a = 507

Let nth term is equal is to zero

a + (n−1)d = 0

507 − (n−1) = 0 (By putting value of a and d)

507 − n + 1 = 0

508 − n = 0

n = 508

So 508th term of AP is zero.

22. Question

If $\frac{1}{x+2}, \frac{1}{x+3}, \frac{1}{x+5}$ are in A.P. Then, x =

A. 5

B. 3

C. 1

D. 2

Answer

Here $\frac{1}{x+2}, \frac{1}{x+3}, \frac{1}{x+5}$ are in A.P

so $\frac{1}{x+3} - \frac{1}{x+2} = \frac{1}{x+5} - \frac{1}{x+3}$

$$\frac{(x+2)-(x+3)}{(x+3)(x+2)} = \frac{(x+3)-(x+5)}{(x+5)(x+3)}$$

$-1/(x+3)(x+2) = -2/(x+5)(x+3)$

1/2 = (x + 2)/(x + 5)

x + 5= 2x + 4

x= 1

23. Question

The nth term of an A.P., the sum of whose n terms is S_n, is

A. $S_n + S_{n-1}$

B. $S_n - S_{n-1}$

C. $S_n + S_{n+1}$

D. $S_n - S_{n+1}$

Answer

The sum of n terms of an A.P is given by S_n

$S_n = n/2\ (2a + (n-1)\ d)$

If the sum of n terms is given that is S_n is given then the n^{th} term is given by the formula

$T_n = S_n - S_{n-1}$

Where S_{n-1} is sum of the $(n-1)^{th}$ term of the A.P.

24. Question

The common difference of an A.P., the sum of whose n terms is S_n, is

A. $S_n - 2S_{n-1} + S_{n-2}$

B. $S_n - 2S_{n-1} - S_{n-2}$

C. $S_n - S_{n-2}$

D. $S_n - S_{n-1}$

Answer

a_n is the n^{th} term of an A.P and a_{n-1} is the $(n-1)^{th}$ term of an A.P,

d = common difference, S_n = sum of n terms of an A.P

$d = a_n - a_{n-1}$

But $a_n = S_n - S_{n-1}$

And $a_{n-1} = S_{n-1} - S_{n-2}$

So $d = S_n - S_{n-1} - (S_{n-1} - S_{n-2})$

$d = S_n - 2\ S_{n-1} + S_{n-2}$

25. Question

If the sums of n terms of two arithmetic progressions are in the ratio $\frac{3n+5}{5n+7}$, then their nth terms are in the ratio

A. $\frac{3n-1}{5n-1}$

B. $\frac{3n+1}{5n+1}$

C. $\frac{5n+1}{3n+1}$

D. $\frac{5n-1}{3n-1}$

Answer

The sums of n terms of two A.P`s are in ratio $\frac{3n+5}{5n+7}$

Let a_1 and d_1 be the first term and common difference of the first A.P, respectively. Similarly let a_2 and d_2 be the common difference of the second A.P, respectively.

According to the given condition

$$\frac{2a_1+(n-1)d_1}{2a_2+(n-1)d_2} = \frac{3n+5}{5n+7}$$

$$\frac{\frac{2a_1+(n-1)d_1}{2}}{\frac{2a_2+(n-1)d_2}{2}} = \frac{3n+5}{5n+7}$$

$$\frac{a_1+\frac{(n-1)d_1}{2}}{a_2+\frac{(n-1)d_2}{2}} = \frac{3n+5}{5n+7}$$

Now the Nth term is given by a + (N−1) d

Equating the coefficients

$$\frac{(n-1)}{2} = N-1$$

⇒ n = 2N – 1

$$\frac{a_1+(N-1)d_1}{a_2+(N-1)d_2} = \frac{3(2N-1)+5}{5(2N-1)+7} = \frac{6N+2}{10N+2} = \frac{3N+1}{5N+1}$$

26. Question

If S_n, denote the sum of n terms of an A.P. with first term a and common difference d such that $\frac{S_x}{S_{kx}}$, is independent of x, then

A. d=a

B. d=2a

C. a = 2d

D. d = – a

Answer

Given AP in which First term = a, Common difference = d, Number of terms = n

And S_n denotes the sum of n terms

So $S_n = \frac{n}{2}(2a + (n-1)d)$

$$S_{kx} = \left(\frac{kx}{2}\right)\{2a + (kx-1)d\}$$

and $Sx = \left(\frac{x}{2}\right)\{2a + (x-1)d\}$

Now,

$$S_{kx} = \frac{\left(\frac{x}{2}\right)\{2a + (x-1)d\}}{\left(\frac{kx}{2}\right)\{2a + (kx-1)d\}}$$

$$= \frac{[\{2a + (x-1)d\}]}{k\{2a + (kx-1)d\}]}$$

$$= \frac{[2a + d - d]}{[k\{2a + kxd - d\}]}$$

$$= \frac{[(2a - d) + xd]}{[k\{(2a - d) + kxd\}]}$$

If d= 2a

$S_x / S_{kx} = (2 \times a) / (k^2 \times 2a)$

$= 1/k^2$

So S_x /S_{kx} is independent of x if d = 2a

27. Question

If the first term of an A.P. is a and nth term is b, then its common difference is

A. $\frac{b-a}{n+1}$

B. $\frac{b-a}{n-1}$

C. $\frac{b-a}{n}$

D. $\frac{b+a}{n-1}$

Answer

'a' is the first term of the A.P,

'b' is the (n)th term and 'd' is the common difference of the A.P.

Then, we have b = a + (n−1) d

= a + (n + 1)d

Hence, 'd' = $\frac{(b-a)}{(n-1)}$

28. Question

The sum of first n odd natural numbers is

A. 2n – 1

B. 2n + 1

C. n^2

D. n^2 – 1

Answer

The sum of the first n odd numbers forms an arithmetic progression with first term equal to 1 and the last term equal to 2(n−1) + 1

The formula for an arithmetic progression of n terms with first term a1 and last term a_n is

$\frac{n}{2}(a_1 + a_n)$

Substituting these values, a_1=1, a_n= 2(n−1) + 1

$\Rightarrow$ n (1 + 2n−2 + 1)2 = n (2n) 2= n^2

29. Question

Two A.P.'s have the same common difference. The first term of one of these is 8 and that of the other is 3. The difference between their 30^{th} terms is

A. 11

B. 3

C. 8

D. 5

Answer

for 1^{st} A.P a = 8 and d= common difference = d

The nth term of an A.P is given by the formula

a_n= a + (n−1) d

The required difference is

= 8 + (30 − 1) d − 3 − (30 −1) d

= 5

30. Question

If 18, a, b, −3 are in A.P., the a + b =

A. 19

B. 7

C. 11

D. 15

Answer

Given 18, a, b, −3 are in A.P

a− 18 = −3 − b

Because the common difference of the A.P. always remains the same

a + b = 15

31. Question

The sum of n terms of two A.P.'s are in the ratio 5n + 9 : 9n + 6. Then, the ratio of their 18th term is

A. $\frac{176}{321}$

B. $\frac{178}{321}$

C. $\frac{175}{321}$

D. $\frac{176}{321}$

Answer

Let a1 and a2 be the first terms of the two A.P. and d1 and d2 be the common difference of the two A.P

According to the given condition,

$$\frac{\text{Sum of } n \text{ terms of first A.P.}}{\text{Sum of } n \text{ terms of second A.P.}} = \frac{5n+4}{9n+6}$$

$$\Rightarrow \frac{\frac{n}{2}\left[2a_1+(n-1)d_1\right]}{\frac{n}{2}\left[2a_2+(n-1)d_2\right]} = \frac{5n+4}{9n+6}$$

$$\Rightarrow \frac{2a_1+(n-1)d_1}{2a_2+(n-1)d_2} = \frac{5n+4}{9n+6} \qquad ...(1)$$

Substituting n= 35 in......... (1)

$$\frac{2a_1+34d_1}{2a_2+34d_2} = \frac{5(35)+4}{9(35)+6}$$

$$\Rightarrow \frac{a_1+17d_1}{a_2+17d_2} = \frac{179}{321} \qquad ...(2)$$

$a_n = a + (n-1)\, d$

$$\frac{18^{th} \text{ term of first A.P.}}{18^{th} \text{ term of second A.P.}} = \frac{179}{321}$$

Or 179: 321

32. Question

If $\frac{5+9+13+\ldots \text{ to n terms}}{7+9+11+.....\text{to}(n+1) \text{ terms}} = \frac{17}{16}$, then n =

A. 8

B. 7

C. 10

D. 11

Answer

$$\frac{5+9+13+\ldots \text{ to n terms}}{7+9+11+\ldots\ldots\text{to}(n+1)\text{ terms}}=\frac{17}{16}$$

Now, Sum of n terms

$\Rightarrow S_n=\frac{n}{2}[2a_1+(n-1)d]$

$=\frac{n}{2}[2\times 5+(n-1)4]$

= n[5 + 2n−2]

=n[3 + 2n]

Now,

$\Rightarrow S_{n+1}=\frac{n+1}{2}[2a_1+(n+1-1)d]$

$=\frac{n+1}{2}[2\times 7+n(2)]$

= (n + 1)[7 + n]

$$\frac{S_n}{S_n+1}=\frac{17}{16}$$

$\Rightarrow \frac{n[3+2n]}{(n+1)(7+n)}=\frac{17}{16}$

On cross multiplying we get,

16n[3 + 2n] = 17n + 17[7 + n]

$\Rightarrow 48n + 32n^2 = 119n + 17n^2 + 119 + 17n$

$\Rightarrow 48n + 32n^2 = 136n + 17n^2 + 119$

$\Rightarrow 15n^2 - 88n - 199 = 0$

$\Rightarrow n = \frac{105}{15} = 7$

33. Question

The sum of n terms of an A.P. is $3n^2 + 5n$, then 164 is its

A. 24th term

B. 27th term

C. 26th term

D. 25th term

Answer

$S = 3n^2 + 5nS_1 = a_1 = 3 + 5 = 8S_2 = a_1 + a_2 = 12 + 10 = 22 \Rightarrow a_2 = S_2 - S_1 = 22 - 8 = 14S_3 = a_1 + a_2 + a_3 = 27 + 15 = 42 \Rightarrow a_3 = S_3 - S_2 = 42 - 22 = 20$∴ Given AP is 8,14,20,.....Thus a = 8, d = 6Given t_m = 164.164 = [a + (n −1)d]164 = [(8) + (m −1)6]164 = [8 + 6m − 6]164 = [2 + 6m]162= 6mm = 162 / 6.∴ m = 27.

34. Question

If the nth term of an A.P. is 2n + 1, then the sum of first n terms of the A.P. is

A. n (n − 2)

B. n (n + 2)

C. n (n + 1)

D. n (n −1)

Answer

a_n= a + (n−1) d = 2n + 1 (given)....................(1)

S_n= n/2 (2a + (n−1) d)

By putting n=1 in (1)

a=3

similarly a_2= 5

a_3= 7

d= common difference = $a_2 - a_1$= 2

S_n = n/2 (6 + (n−1) 2)

= n/2(2n + 4)

= n (n + 2)

35. Question

If 18th and 11th term of an A.P. are in the ratio 3 : 2, then its 21st and 5th terms are in the ratio

A. 3: 2

B. 3: 1

C. 1 :3

D. 2 : 3

Answer

As 18^{th} term : 11^{th} term ratio is 3:2

18^{th} term is a + 17d and 11^{th} term is a + 10d

$(a + 17d)/(a + 10d) = 3/2$

3a + 30d = 2a + 34d

a=4d...............(1)

21^{st} term is a + 20d = 24d (putting value of a from (1))

And 5^{th} term is a + 4d= 8d (putting value of a from (1))

Ratio of 21^{st} term : 5^{th} term is

$24d/8d = 24/8 = 3/1$ = 3:1

36. Question

The sum of first 20 odd natural numbers is

A. 100

B. 210

C. 400

D. 420

Answer

Here A.P is 1,3, 5, ...

a= 1, d= 2 and n = 20

$S_n = n/2$ (2a + (n−1) d)

S_n = 20/2 (2 x 1 + (19) 2)

S_n = 10 (2 + 38)

S_n= 400

37. Question

The common difference of the A.P. is $\frac{1}{2q}, \frac{1-2q}{2q}, \frac{1-4q}{2q}, \ldots$, is

A. −1

B. 1

C. q

D. 2q

Answer

Here A.P = $\frac{1}{2q}, \frac{1-2q}{2q}, \frac{1-4q}{2q}, \ldots$

d= a2− a_1

$d = \frac{1-2q}{2q} - \frac{1}{2q}$

d= $\frac{1-2q+1}{2q}$

d= −1

38. Question

The common difference of the A.P. $\frac{1}{3}, \frac{1-3b}{3}, \frac{1-6b}{3}, \ldots$ is

A. $\frac{1}{3}$

B. $-\frac{1}{3}$

C. −b

D. b

Answer

A.P = $\frac{1}{3}, \frac{1-3b}{3}, \frac{1-6b}{3}, \ldots$

d = common difference = $a_2 - a_1$

= $(1-3b)/3 - 1/3$

d= $\frac{1-3b-1}{3}$

d= −b

39. Question

The common difference of the A.P. $\frac{1}{2b}, \frac{1-6b}{2b}, \frac{1-12b}{2b}, \ldots$ is

A. 2b

B. −2b

C. 3

D. −3

Answer

Here A.P is $\frac{1}{2b}, \frac{1-6b}{2b}, \frac{1-12b}{2b}, \ldots$

d= $a_2 - a_1$

d= $\frac{1-6b}{2b} - \frac{1}{2b}$

d= $\frac{1-6b-1}{2b}$

d= −3

40. Question

If k, 2k −1 and 2k + 1 are three consecutive terms of an AP, the value of k is

A. − 2

B. 3

C. − 3

D. 6

Answer

Here A.P = k, 2k −1, 2k + 1

Since the numbers are in A.P their common difference (d) should be same

d=$a_2 - a_1$ = $a_3 - a_2$

2k−1−k = 2k + 1− (2k −1)

k− 1 = 2

k = 3

41. Question

The next term of the A.P. $\sqrt{7}, \sqrt{28}, \sqrt{63}, \ldots$

A. $\sqrt{70}$

B. $\sqrt{84}$

C. $\sqrt{97}$

D. $\sqrt{112}$

Answer

A.P is $\sqrt{7}, \sqrt{28}, \sqrt{63}, \ldots$

The simplified A.P is $\sqrt{7}$, $2\sqrt{7}$ $3\sqrt{7}$

d= $2\sqrt{7} - \sqrt{7} = \sqrt{7}\,(2-1) = \sqrt{7}$

Next term of A.P means the fourth term = a_4

$a_4 = a + (n-1)\,d$

$a_4 = \sqrt{7} + 3\sqrt{7}$

$= 4\sqrt{7}$

$= \sqrt{112}$

42. Question

The first three terms of an A.P. respectively are 3y − 1, 3y + 5 and 5y + 1. Then, y equals

A. −3

B. 4

C. 5

D. 2

Answer

A.P here is 3y – 1, 3y + 5 and 5y + 1.

So d= common difference = $a_2 - a_1 = a_3 - a_2$

3y + 5 – (3y−1) = (5y + 1) – (3y + 5)

6 = 2y – 4

10 / 2 = y

Y = 5